Applying Critical Mathematics Education: An Introduction

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Abstract

In this introductory chapter, we first set out our broad characterisation of critical mathematics education, drawing on contemporary issues including, for example, global climate change and rapid societal challenges. Critical mathematics education is driven by urgent, complex questions; is interdisciplinary; is politically active and engaged; is democratic; involves critique; and is reflexive and self-aware. This perspective leads us to argue for the necessity of critical mathematics education, for which we summarise three significant traditions derived from Freire, Foucault, and the Nordic School. Finally, we provide an overview and discussion of the contributions to this volume, and show how they apply critical mathematics education in unique ways that relate to the six previously described features of this approach. We conclude by reiterating the urgent necessity of applying critical mathematics education.

Keywords

critical mathematics education – Freire – Foucault – Skovsmose – mathematics – climate change – race and racism – ethnomathematics – Indigenous perspectives

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This is all wrong. I shouldn't be up here. I should be back in school on the other side of the ocean. Yet you all come to us young people for hope. How dare you! You have stolen my dreams and my childhood with your empty words. And yet I'm one of the lucky ones. People are suffering. People are dying. Entire ecosystems are collapsing. We are in the beginning of a mass extinction, and all you can talk about is money and fairy tales of eternal economic growth. How dare you!

[...]

The popular idea of cutting our emissions in half in 10 years only gives us a 50% chance of staying below 1.5 degrees [Celsius], and the risk of setting off irreversible chain reactions beyond human control. Fifty per cent may be acceptable to you. But those numbers do not include tipping points, most feedback loops, additional warming hidden by toxic air pollution or the aspects of equity and climate justice. They also rely on my generation sucking hundreds of billions of tons of your CO₂ out of the air with technologies that barely exist.

So, a 50% risk is simply not acceptable to us—we who have to live with the consequences. To have a 67% chance of staying below a 1.5 degrees global temperature rise—the best odds given by the [Intergovernmental Panel on Climate Change]—the world had 420 gigatons of CO₂ left to emit back on January 1st, 2018. Today that figure is already down to less than 350 gigatons. How dare you pretend that this can be solved with just “business as usual” and some technical solutions? With today’s emissions levels, that remaining CO₂ budget will be entirely gone within less than 8½ years.

There will not be any solutions or plans presented in line with these figures here today, because these numbers are too uncomfortable. And you are still not mature enough to tell it like it is. You are failing us. (Greta Thunberg’s address at the United Nations Climate Action Summit in New York City, 23 September 2019)¹

We wrote the first draft of this introduction in the same week that Greta Thunberg made her speech at the UN Climate Action meeting in New York in the autumn of 2019. Millions of young people worldwide were demonstrating with the goal of making our state leaders take action and follow the Paris Agreement on climate change. Thunberg’s speech, partly quoted above, is a powerful call for change. It also involves a lot of applied mathematics. She refers to probabilities describing future global temperature change, as well as notions like tipping points and feedback loops. She refers to carbon budgets and projections for the likely timelines for humans to burn through that budget. Thunberg relates this mathematics to her values and her rage; she connects the mathematics of climate change to the responsibilities of leaders.

Greta Thunberg began her climate activism as a secondary school student when, instead of attending classes, she sat outside the Swedish parliament to protest at government inaction on climate change. Since that time, she has become the figurehead of worldwide youth-led demonstrations calling for stronger, faster action on climate change. As such, she is seen as a remarkable individual—a student who skipped school to change the world.

What would or could mathematics education be like if we took Greta Thunberg as its inspiration? It would be driven by urgent, complex questions that

connect mathematics with science, geography, languages, and other curriculum subjects. It would be politically active and engaged, involving direct interaction with political leaders and institutions. It would be democratic in its orientation, promoting access to significant current information, debate about possible responses, and participation. This mathematics education would involve critique, using mathematics to interrogate complex problems, to question and to challenge. It would also be reflexive and self-aware, critiquing the role of education and of mathematics itself in contributing to problems like climate change. Thunberg, it should be noted, did not neglect her school assignments. It's just that school work was insufficient to change the world.

This vision of mathematics education is, broadly, a good example of what we mean by *critical mathematics education*. Over the past decades, different versions of critical mathematics education have emerged, drawing on different theoretical sources and traditions. While these different versions each have their distinctive aspects, they generally all share these common underlying features: in summary, critical mathematics education is *driven by urgent, complex questions*; is *interdisciplinary*; is *politically active and engaged*; is *democratic*; *involves critique*; and is *reflexive and self-aware*.

There is no shortage of urgent, critical problems that mathematics education can and should engage with. During the time it has taken to complete this book, we have seen the COVID-19 pandemic, widespread forest fires in many parts of the world, more evidence of how plastic particles have contaminated every part of the web of life, Black Lives Matter protests, and several election campaigns notable for “fake news.” These phenomena are all a matter of life and death, all have a clear political dimension, all are interdisciplinary in nature, and all involve mathematics in different ways. There is plenty to be done with critical mathematics education.

There is now a well-established critical mathematics education research literature, as well as a growing popular literature. In the next part of this introductory chapter, we provide an overview of this work, first highlighting recent trends in the public understanding of mathematics, before offering an overview of significant early contributions, key concepts and principles in critical mathematics education. These contributions explore how mathematics teaching, curriculum and assessment is often implicated in social, economic and environmental injustice, either through the inequities of schooling, or through the content and nature of the mathematics curriculum.

Schooling is, of course, a central organising institution in the structuring of contemporary society. How does mathematics in school contribute to social-economic, racial or gender inequities? The mathematics curriculum, as proposed, taught and lived, paints a particular portrait of mathematics and of society. Does this curriculum prepare docile workers for an increasingly

polarised economic system, or does it prepare future citizens who are able to critique the conditions of their lives and work for change? More broadly, how do popular discourses about mathematics contribute to problematic features of the organisation of our society? And how does our own activity as mathematics education researchers maintain or dismantle inequitable or problematic structures?

Research in critical mathematics education has sought to address these kinds of questions. We argue that much of this work is theoretical in nature, doing the important intellectual work of shaping the terrain, through the elaboration of concepts, principles and ideas. One of our motivations for proposing this book was to offer a collection of writing that shows how these concepts, principles and ideas can be *applied*, in school classrooms, teacher education classrooms, in curriculum development or in research projects. In so doing, we wanted to extend the nature and scope of what applying critical mathematics education can do; and we also sought to include scholars applying ideas from critical mathematics education in contexts that have rarely appeared under this heading in the past. In the last part of this chapter, we discuss some different forms of *applying* critical mathematics education and explain how they are illustrated by the chapters of this book.

1 The Growing Relevance of Critical Mathematics Education

Before examining the scholarly literature falling under the general heading of critical mathematics education, we first draw attention to the increasing popular awareness of how mathematics sometimes contributes to inequality or social injustice. More and more books and articles are appearing that draw attention to the role of mathematics in our society. These are not simply popular mathematics texts, designed to explain how probability or geometry are applied in contemporary society. A book like Nate Silver's *The Signal and the Noise* (2012), for example, makes a valuable contribution to explaining the application of probability in topics as diverse as baseball, the weather, political polling and election predictions. This book is critical in the sense that it contributes to an understanding of the application of Bayesian statistics in commonly consumed information. However, it is not critical in our stronger sense of critiquing the implicit biases and assumptions embedded in this mathematics.

Several other recent popular books offer examples of this more critical take on the role of mathematics in society. The central argument of Caroline Criado Perez's *Invisible Women* (2019), for example, is that a lack of gender-specific

data in many aspects of society, including the economy, healthcare, design and architecture, employment practices, and government welfare programmes, results in systemic biases against women. To give one example, car safety standards are, according to Criado Perez, almost entirely based on a standard male norm, including a standard crash test dummy modelled on a “typical” 70 kg man. As a result, the majority of car safety data is not valid for women drivers. And it turns out that car safety features are not as effective for women, resulting in a higher proportion of injuries and deaths when women drivers are involved in road traffic accidents. Another example of this male bias has recently been highlighted in the UK, where the personal protective equipment necessary for healthcare workers fighting the COVID-19 pandemic was reportedly “designed for a 6 foot 3 inch bloke built like a rugby player.”² Since approximately 80% of healthcare workers are women, the equipment was, incomprehensibly, designed for a male minority. For women healthcare workers, this equipment would often not fit correctly, thereby putting their own health and lives at risk. Criado Perez’s book is an excellent resource for critical mathematics education. It is driven by a complex and persistent problem—that of a structural male bias in much economic and scientific activity. It challenges leaders and organisations to make structural changes, and clearly highlights how mathematics and science do not magically ensure an objective gender-aware approach to problem-solving.

In *Weapons of Math Destruction*, Cathy O’Neil (2017), writes about the inequality built into numerous socially significant algorithms, in relation to domains like crime prevention, insurance, and healthcare. In one chapter, for example, she shows how algorithms designed to make policing more efficient in some cities in the United States predict crime hotspots and so determine where police resources are allocated. These hotspots coincide with low-income (which in the US are often majority Black) neighbourhoods which are then subject to intensive policing methods. These methods in turn can result in a higher than average proportion of arrests for low-level infractions, so that residents in these neighbourhoods are more likely to have criminal records. Part of the bias comes from the fact that low-level infractions also occur in affluent neighbourhoods (including so-called “white collar” crime), but since police are not deployed in these neighbourhoods, criminal records are not built up. This chain of circumstances is driven by mathematics but results in the perpetuation of a form of inequality and, indeed, deadly structural racism, in urban policing.

In *Algorithms of Oppression*, Safiya Umoja Noble (2018) investigates structural biases in internet search algorithms with respect to race. She shows, for example, how entering into Google “why are black women so ...” results

in words like angry, loud and mean, while similar searches for white women result in the words pretty and beautiful. Again, this bias is a result of complex mathematical algorithms, is, one assumes, not deliberate, but nevertheless perpetuates problematic racial and gender stereotypes. Both O'Neil's and Noble's books are resources for critical mathematics education. They make visible and critique how mathematics is part of the fabric of society, through the use of algorithms that encode human biases, blind spots and prejudices.

Hans Rosling, through his posthumous book *Factfulness: Ten Reasons We're Wrong About the World and Why Things Are Better Than You Think*, challenges our preconceptions and misconceptions (Rosling, Rosling Rönnlund, & Rosling, 2018). He asks why so many people, including Nobel laureates and other researchers, journalists, politicians and “ordinary” people, get some numbers so wrong on pressing issues such as poverty, pandemics and climate change. He shows how divisive ideas (separating *us* from *them*), the huge influence of the media and social media (telling us what to fear), and our perception of progress (seeing how things are getting worse rather than better) may hinder clear conversations and knowledge development. He invites us to think about, for example, the “gap instinct” (p. 31), or what we understand as “gap mathematics”: the tendency to divide everything into two opposites, or dichotomies, or binary thinking. For example, consider comparisons of mean values, and the impact such comparisons might have on our worldview, when not also analysing the equally important distribution of data. Only looking at mean values gives very different results from analysing the distribution of women's and men's results in mathematical SAT-tests, for example. Rosling's point is that this “us and them” construct, or the “gap instinct,” is not borne out by facts. It usually originates in large comparisons where the differences of the means and averages between compared labelled groups might not be as big as these means or averages indicate when distribution is also analysed.

There is a powerful political rhetoric around the “gap instinct” which we believe clearly connects with critical mathematics education. For example, the phrase “the growing gap,” so frequently used by journalists in the media and by political leaders in their rhetoric, is an example of “gap” language where mean and average numbers are used for comparisons instead of measures of distribution, such as standard deviation or variance. We believe it is important to talk about and teach about these consequences and other mathematically oriented rhetorical language, such as the “generalisation instinct” (Rosling, Rosling Rönnlund, & Rosling, 2018, p. 173), the “perspective instinct” (p. 221) and the “urgency instinct” (p. 265). This is an important conversation to have specifically in mathematics education, since arguments with numbers are rhetorically powerful.

The fact that books like these have appeared in recent years is a positive development. There seems to be a greater awareness in society and in popular culture that mathematics, and in particular, the use of algorithms, are not simply neutral tools for enhancing efficiency in information systems. Mathematics, the communication of mathematics, and mathematics education can have unintended but very real consequences for people's lives. The question that follows for educators from this awareness is what, if anything, should mathematics teachers do about it? Should we, as teachers of mathematics, or educators of teachers of mathematics, address this dimension of mathematics in our work? This is precisely where critical mathematics education has something to offer.

2 Three Traditions of Critical Mathematics Education

Most research in mathematics education is concerned with questions of teaching, learning, assessment, curriculum, teacher education, and so on: practical problems arising from mathematics education in classrooms. It seeks to answer questions about how children or adults learn mathematics, how best to teach mathematics, how to prepare teachers to teach mathematics and how to assess what students have learned. This work is, of course, entirely reasonable and important. Nevertheless, there is a danger that this kind of focus is too narrow. For most mathematics educators, mathematics is not simply a set of procedures to be applied, or a set of facts to be memorised. Mathematics is a way of thinking and understanding the world. More particularly, mathematics is a human activity and reflects human concerns. It also, therefore, reflects human biases, blind spots and structures of dominance and oppression. A long-standing strand of research has adopted socio-cultural, socio-political and critical perspectives (for example, Atweh, Forgasz, & Nebres, 2001; Gutiérrez, 2013; Valero & Zevenbergen, 2004). This work has often been focused on the “hidden curriculum” of mathematics classes, the role of mathematics in the creation of a particular kind of social and economic organisation, such as that which has resulted in the climate crisis, and the need for students to learn about the role of mathematics in structuring their society and their lives.

In this section, we restrict ourselves to reviewing work that fits our characterisation of critical mathematics education research—that is, research that examines ways in which mathematics education can be driven by urgent, complex questions; is interdisciplinary; is politically active and engaged; is democratic; involves critique; and is reflexive and self-aware. We introduce three broad critical mathematics education traditions: Freirean, Foucauldian and the Nordic School. This characterisation is, of course, a simplification and the

different traditions (and others besides) have certainly influenced each other in important ways, sometimes explicitly, but often implicitly. These different traditions share similar objectives but draw on different theoretical ancestors. This organisation is helpful for making sense of the different strands of thought influencing the field of critical mathematics education, but should definitely not be seen as creating discrete silos of work.

The first tradition we will describe is inspired by the work of Freire, focused on the value of mathematics for “reading the world.” Freire’s work was itself directly inspired by Marx’s analysis of the structural basis within capitalism of the oppression of the working class and can be understood as an extension of Marx’s ideas into the realm of pedagogy. In particular, Freire’s emphasis on the importance of consciousness can be traced to Marx so that for Freire, a critical, transformative education must be designed around consciousness raising, with a particular focus on literacy (Lake & Kress, 2013). If, with Marx, Freire believed that only the oppressed can break the bonds of their oppression, then literacy was the key and the focus of his *Pedagogy of the Oppressed* (Freire, 2007).

Freire’s ideas have particularly influenced North American approaches to research in critical mathematics education. In this tradition, mathematics is understood as being used not only for doing commerce, engineering, science, and so on, but also for conducting critical analyses of society, as a way of reading and transforming the world. The role of mathematics education is, therefore, to enable students, and particularly students who are marginalised by society in different ways, to use mathematics as a tool for their own empowerment. Marilyn Frankenstein was an early proponent of Freire’s work in mathematics, publishing an article in 1983 with the title “Critical mathematics education: an application of Paulo Freire’s epistemology.” In this work, she draws out the important link between epistemology and emancipation or social transformation in relation to mathematics teaching:

Freire’s theory compels mathematics teachers to probe the nonpositivist meaning of mathematical knowledge, the importance of quantitative reasoning in the development of critical consciousness, the ways in which math anxiety helps sustain hegemonic ideologies, and the connections between our specific curriculum and the development of critical consciousness. In addition, his theory can strengthen our energy in the struggle for humanization by focusing our attention on the interrelationships between our concrete daily teaching practice and the broader ideological and structural context. (Frankenstein, 1983, p. 324)

The key point here is that it is insufficient to think of mathematics as a tool for understanding the world and therefore potentially as a tool for understanding

inequality or oppression. Drawing on Freire's ideas, Frankenstein argues that we need to change how we think about the nature of mathematical knowing. Mathematics is a human activity and as such reflects human relations, including oppressive or hegemonic relations embedded in our social structures. Mathematics teaching must therefore address the social basis of mathematical knowledge and its implication in the structure of society, in order to empower students to challenge oppression.

This kind of approach has slowly been taken up around the world, but particularly in the Americas. Most notably, Gutstein's *Reading and Writing the World With Mathematics: Toward a Pedagogy for Social Justice* (2006) describes his work as a high school mathematics teacher in Chicago in the 1990s and into the 2000s. The Freirean influence is clear from the beginning:

Students need to be prepared through their mathematics education to investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts—that is, to “read and write the world” with mathematics ... to read the world is to understand the socio-political cultural-historical conditions of one's life, community, society, and world; and to write the world is to effect change in it. (p. 4)

As an illustration of this approach, the book begins with Gutstein's account of teaching mathematics in September 2002, one year after the 9/11 attacks on New York. He describes how, over the course of the following days, he explores students' questions about the attacks and constructs mathematics problems, such as one about the number of full four-year university scholarships that could be paid for with the cost of one B2 stealth bomber. This example illustrates how students can use mathematics to read the world, and potentially, like Greta Thunberg, to effect change in it. More implicit is the epistemological status of mathematics highlighted by Frankenstein: mathematics teaching in this way also challenges students' relationship with mathematics. Students are not passive consumers of mathematical information and methods; they are active in seeking, interpreting and critiquing mathematical information and its application.

The second tradition of critical mathematics education research we describe draws on the work of Foucault. In this tradition, the focus is on mathematics as a discourse that plays a role in the organisation of human affairs. That is, the uses of mathematics, or references to mathematics, have various effects, many of which may be largely unnoticed or even invisible. From this perspective, the teaching and learning of mathematics is not simply a process of transmitting facts and procedures from teachers to students. Underlying the teaching and learning process are assumptions about the nature of rationality, of students,

of teachers and of society, assumptions that may not be apparent to any of the participants. Mathematical discourses, for example, can construct particular kinds of categories in particular orders. To give one example, the idea of “normal,” such as normal height, normal personality or normal behaviour, only arose with the emergence of modern statistics in the nineteenth century (Hacking, 1990). In similar vein, discourses of mathematics education construct learners and teachers in particular ways, as having particular roles and as performing particular functions, such as, for instance, the idea that mathematics is an innate talent seen in a few socially awkward boys and men.

Walkerdine's (1988) critical analyses of the mathematics education of young children is an important early example of a Foucauldian critical mathematics education study. Walkerdine's wide-ranging work drew explicitly on Foucault's ideas to show how ideas about learning and about mathematics produced particular kinds of children and particular kinds of rationality. In particular, she critiqued the then popular Piagetian perspective on children's mathematical learning and development, showing how it depended on particular assumptions about children and about mathematics that, in fact, reflected discourses of modernity, rationality and what it means to be human. Such theories, she argued, assume development to be a progressive, natural process that would, in “normal” children, result in a developed rationality of deductive reasoning and control of one's world. A key point here is that there is nothing “natural” about these assumptions or about this kind of rationality. Walkerdine also showed how the discourses of early childhood mathematics classrooms were strongly gendered, producing differences in the construction of girls and boys in relation to mathematics.

We call the third tradition of critical mathematics education the Nordic School as it was initiated by Scandinavian researchers. An early contribution to this strand of thinking came from Stig Mellin-Olson's (1987) book *The Politics of Mathematics Education*, in which he made two important points still relevant today. First, he noted that the diverse cultural practices in which different forms of mathematics arise (e.g., housebuilding, playground games) imply a political dimension to mathematics education. This political dimension is related to the relationships of dominance and resistance between the various cultures in which these cultural practices are embedded. Second, Mellin-Olsen highlighted the “the ideological content of mathematical models: what is being left out in the mathematical model, and which economic, physical or social theory leads to the relationships which are mathematised?” (p. 203).

These ideas were taken up and developed by Ole Skovsmose. In *Towards a Philosophy of Critical Mathematics Education*, Skovsmose (1994) sets out a detailed argument about how mathematics shapes modern society, often

through technology, in often invisible ways. As a result, Skovsmose argues that it is not sufficient for children to only learn about how to do or use mathematics. They also need to learn about, and how to critique, the role of mathematics in their lives, in society and not the least, as a crucial part of the technology of economic development.

The Nordic School has developed ideas from within mathematics education, with a particular focus on how to change “traditional” mathematics classroom teaching towards a more student-centred environment in which societal challenges can be addressed, in which the role of mathematics in society can be pin-pointed, and in which critique of mathematics and its applications is allowed and even expected. This shift is accompanied by a reconceptualisation of the role of students in mathematics classrooms from a place of silence, individual calculations and mathematical exercises to a space of dialogue, reflection and critique (see Skovsmose, 2001a, 2009, 2012; Skovsmose & Nielsen, 1996).

These three broad traditions of critical mathematics education have different theoretical sources and slightly different foci. The Freirian tradition draws on Marxian theory and focuses on reading and writing the world with mathematics. The second tradition draws on Foucault’s post-structuralist theory and focuses on mathematics in relation to discourses. The third tradition focuses more clearly on mathematics and mathematics learning, as developed most notably by Skovsmose. Despite their differences, all three seek to interrogate the world with mathematics, and mathematics with the world, through focusing on significant issues, often relating to injustice, oppression and critiques of contemporary society. All three traditions lend themselves to democratic values and political participation. And all three subscribe to a more expansive, and hence interdisciplinary, view of mathematics as being about more than a collection of facts, concepts and procedures.

In recent years, research in critical mathematics education has diversified and has made connections and synergies with other critical traditions in mathematics education or in educational research more broadly. Critical mathematics education researchers have engaged, for example, with ethnomathematics (Knijnik, 2007; Powell & Frankenstein, 1997), with Indigenous and decolonising perspectives (e.g., Gutiérrez, 2017; Parra-Sanchez, 2017), with critical pedagogy (Appelbaum, 2008), with the politics of post-colonial contexts (Vithal, 2003) or with gender studies (see Mendick, 2006). These different theoretical and methodological traditions have been developed extensively in mathematics education research and classrooms, and there are currently examples of how these theories can be applied in classrooms to better understand the role and power of mathematics in society and in education.

3 Applying Critical Mathematics Education: The Contribution of This Book

We see this book as distinct from previous collections of writing about critical mathematics education due to its focus on *applying* critical mathematics education. By “applying,” we do not simply mean a teaching activity or programme. We seek to illustrate and explain how critical mathematics education can be used to understand the role of mathematics in mathematics classrooms, teacher education programmes or in critical analyses of specific issues. The book therefore extends the ways in which ideas in critical mathematics education have been applied. It shows how critical mathematics education can be applied to a wide range of topics, including topics not previously addressed (e.g., environmental sustainability, obesity). Our aim was to collect examples of applying the theoretical apparatus of critical mathematics education to understand a wide range of contemporary social phenomena in new ways. Chapters apply critical mathematics education to contemporary social challenges that have been under-examined in mathematics education, such as the global economy, peace, racial justice, decolonisation and migration.

The chapters showcase work conducted in different parts of our world, written by researchers with a diversity of mother tongues and cultural backgrounds, conducting their teaching, research and writing in their specific local contexts, but with global relevance and importance. The authors all share a profound belief that mathematics education can make a change in and for the world, through changing children's, students', teachers' and researchers' perceptions of mathematics, mathematics learning and the role of mathematics in society. We wish we could add that we share this belief with politicians, policymakers and curriculum designers, but we fear that this goal has yet to be reached.

We have organised the book in three informal parts. The chapters in the first part address questions about relationships between critical mathematics education, ethnomathematics and mathematics education research as a form of culturally situated research. Hence these chapters interrogate intersections of ethnomathematics, decolonisation and Indigenous ways of knowing. In these chapters, critical mathematics education is applied as dialogue between researchers and community members, between different ways of knowing mathematics, and between different theories. The second part comprises two chapters that focus on the importance of the theoretical and epistemological choices we make in the conduct of our research with respect to the dynamics of race and the necessity of researcher reflexivity. In these chapters, critical mathematics education is applied to an understanding of the research process. The third part focuses on research that seeks to understand the role

of mathematics in organising society, and that works to include this focus in mathematics classrooms. In these chapters, critical mathematics education is applied in critical analyses of the social relevance of mathematics, as well as in mathematics classroom pedagogy. Of course, the chapters in the book could have been organised in many different ways and some chapters would fit well into more than one part, and the themes are interwoven. Nevertheless, they provide some structure for the book and support the reader in navigating between the chapters.

3.1 *Intersections of Critical Mathematics Education, Ethnomathematics, Decolonisation and Indigenous Ways of Knowing*

If critical mathematics education is politically engaged, democratic and reflexive, then its researchers must critically examine the role of their work in perpetuating structures of injustice and oppression, and must ask how their work can contribute to dismantling such structures. This kind of critical reflexivity is hard to do: as Foucault showed, these structures are embedded in our ways of knowing and interacting, and are therefore difficult to dismantle. What is known as mathematics comes from a specific Indo-European tradition that encodes a particular set of habits of mind. The challenge of how to see beyond our particular ways of knowing is particularly apparent in work that seeks to critically engage with ways of knowing that fall outside of this tradition and to do so in ways that are respectful, reciprocal and empowering. Critical mathematics education has perhaps not always put such concerns in the foreground.

Researchers working within ethnomathematics have grappled with some of these challenges for some time (see, for example, Barton, 2008; Pinxten & François, 2011). There has been little work, however, that applies ideas from critical mathematics education to ethnomathematics. The first contribution to this book, by Annica Andersson and David Wagner, explores possibilities for synthesising critical mathematics education and ethnomathematical concerns for teaching and research. Through a discussion of the history of ethnomathematics and of critical mathematics education, they tease out and discuss the intersections between these two areas, and develop a theoretical framework building on these common concerns. The framework is then used to analyse two empirical articles: one by Nutti (2013), who accounts for ethnomathematical research and teaching in an Indigenous Sápmi context; and one by Rubel et al. (2016) who give an account of critical mathematics education in a disadvantaged North American context. With this analysis, the authors demonstrate how the application of ideas from critical mathematics education to ethnomathematics is productive for both approaches. In particular,

critical mathematics education gains a stronger sense of the situatedness of mathematical knowing.

In the chapter by Lisa Lunney Borden, a critical mathematics perspective is explored in relation with Indigenous mathematics education in Atlantic Canada. Using examples from her own 22-year learning journey together with a Mi'kmaw First Nation's community, she shows powerfully that moving towards decolonising mathematics and mathematics education comprises much more than applying mathematics to culturally relevant contexts or artefacts. Lunney Borden situates her text in the stories witnesses have shared about their experiences in Indian Residential Schools,³ now published in reports arising from Canada's Truth and Reconciliation Commission. Hence, we are now starting to understand the impact these schools have had on over 150,000 Indigenous children, spanning over 100 years, and the inter-generational trauma that continues to this day. Lunney Borden lays out the role education has played in "creating inequity, perpetuating stereotypes and silencing Indigenous voices," and makes the clear claim that education is also the vehicle for addressing these wrongs and finding a new way forward. Her examples of critical mathematics education projects include examining diabetes rates in Indigenous communities, language revitalisation, persistent problems with unsafe drinking water and social justice issues. The projects described by Lunney Borden are contextually grounded as they emerged from specific Indigenous communities and were rooted in the stories shared by both Indigenous school teachers and Elders. Her chapter is an example of how critical mathematics education can contribute to the dismantling of oppressive social and economic structures by engaging with critique and decolonisation of mathematics, of education and of mathematics education research.

The third chapter in this part also engages with decolonisation through an Indigenous Colombian perspective on critical mathematics education. Aldo Parra and Paola Valero invite us to the Lomitas community in the Andes mountains, in the region of Tierradentro in Colombia, a village that is only accessible on foot. There, through Parra's field work, they explore the thought-provoking conceptualisation of *propio*. *Propio* is a notion that emerged in Colombian Indigenous education to "think about, characterise and purposefully manage the relationship between community, knowledge and action in face of the life and political struggles of an Indigenous group." *Propio* may be acknowledged as an educational project, revealing decolonising standpoints and aiming to endorse a critical understanding of, for example, both intra- and extra-cultural conflicts and possibilities for a cultural diversity. Parra and Valero are applying critical mathematics education through democratic, participatory, and reflexive methods.

These three chapters are examples of applying critical mathematics education in ways that reflect all three traditions we described above. We see traces of the Freirean tradition, for example, in Parra and Valero's account of how the mathematics is driven by critical issues identified and developed by the people of Lomitas. We see traces of the Foucauldian tradition in the recurring attention to language and to the ordering of language and ways of knowing. In her work, Lunney Borden and her collaborators, for example, seek to re-order both language and ways of knowing through a repositioning of Mi'kmaw language in mathematics. And all three chapters examine how mathematics contributes to social reality in ways that extend beyond the technological contexts often considered in the Nordic tradition.

3.2 *The Dynamics of Race in Critical Mathematics Education Research*

In the second informal part of the book, the critical, reflexive, politically engaged commitments of critical mathematics education are applied to questions of race and racism. It is a matter of serious concern that critical mathematics education researchers do not seem to have engaged deeply with this topic, despite some clear connections. Critical race theory has had a growing influence within mathematics education (e.g., Davis & Jett, 2019; Martin, 2013) and has highlighted structurally racist assumptions within the organisation of mathematics curriculum, schooling and in mathematics education research, particularly in the United States (e.g., Martin, 2013), but critical mathematics education researchers have been slow to extend their work in this direction.⁴ The two chapters in this part invite us to do just that.

Kate le Roux and Sheena Rughubar-Reddy situate their chapter theoretically, socially, culturally and historically in the complex context of South Africa. More specifically, they examine mathematics education in an access programme for prospective medical doctors at the University of Cape Town, in which its racial history impacts what happens today. In particular, the programme is designed to correct the overwhelming disadvantage Black students have had in being admitted to medicine programmes. The authors guide us through the voices of the university's historical structures, the experiences of Matla, a student in the programme, and through voices from student protests and strikes for racial justice. Building on the South African academic Hilary Janks's critical literacy framework and the concepts of access, domination, diversity and design, the authors tease out complex relationships between mathematics, mathematics education and power. Through using southern African theories, they challenge our "northern" epistemologies and theories, and respond to the question, in the place and context of South Africa: "What might a critical mathematics education look like in this extremely complex context at this moment in time?"

The second chapter in this part, by Victoria Hand, Beth Herbel-Eisenmann, Sunghwan Byun, Courtney Koestler and Tonya Bartell, is written in the context of an educational research project on the development of equitable mathematics education systems in the United States. This group of authors share their collaborative encounters, experiences and tensions that arose during a critical reflexive study of Whiteness within the research group, exploring how the collaborative project maintained White institutional space. The analysis phase of the overall project resulted in “insights that came at significant cost to scholars of Colour involved in the analysis” and that hence threatened the project as a whole. They describe the tensions and perpetuation of patterns of racial privilege and dominance that occurred when White scholars in the team pushed the project forward, without taking time to hear and discuss moments of “confusion, doubt or other emotionally-led stopping-points.” The group utilises the notion of “failure” to describe the “systematic ways that ‘best intentions’ become a means for further oppression.”

These two chapters show how critical mathematics education can be applied to questions of racial justice, although both are clearly first steps. Le Roux and Rughubar-Reddy’s chapter shows how mathematics education can act as a racially organised and organising gate-keeper, something that is apparent in many other parts of the world. Similarly to the chapters in the previous part, they question the ordering of ways of knowing about mathematics and mathematics education and seek to invert that ordering by bringing locally relevant perspectives to their analysis. In Hand et al.’s chapter, this ordering extends to the social practices within the research group. Their chapter demonstrates how applying critical mathematics education means reflexively paying attention to the conduct of critical mathematics education research.

3.3 *Understanding the Role of Mathematics in Organising Society*

The last set of chapters focus squarely on the organising role played by mathematics in contemporary society, a role particularly highlighted by Skovsmose (e.g., 1994). Indeed, Skovsmose has provided several specific examples in his writing, including that of airline seat reservation systems that rely (or did so pre-pandemic) on algorithms and overbooking to ensure full flights, with the result that occasionally some passengers would be “bumped” because too many seats had been sold (Skovsmose, 2001b). This idea, and the explanatory concept of “formatting,” in which mathematical models become prescriptive of reality, has an analytic potential that has only recently started to be realised. Indeed, O’Neil’s (2017) book, in effect, sets out a series of examples. The chapters in this part, then, extend these ideas into new areas.

The chapter by Ulrika Ryan, Annica Andersson and Anna Chronaki, describes a multilingual and multicultural school context in Sweden where a 10-year-old student argued that “Mathematics is bad for society.” Inspired by this statement, a small-scale project was created to explore this idea in his exceptionally language diverse, but unilingual (Swedish only) middle school classroom. The young students were challenged to make value-laden critical interconnections between mathematics and society, because in such a language diverse classroom “embracing students’ personal ways of knowing is intertwined with the socially and culturally normative inferences inherent in language(s).” Theoretically, the authors mainly frame their analysis with Wittgensteinian ideas, such as language games and, specifically, the *language game of giving and asking for reasons* (GoGAR) (Brandom, 1998, 2001). The authors argue that the conversations about the idea that “mathematics is bad for society” allowed the young students to see mathematics and society as interconnected and to grasp the political role of mathematics in society, and hence to become critically mathematical literate. The students had to shift from accepting to questioning the strong prevailing discourses about mathematics as neutral or beneficial. This chapter illustrates how critical mathematics education can be applied in a classroom setting.

The second and third chapters in this part look at the connection of mathematics to climate change. Richard Barwell and Kjellrun Hiis Hauge’s chapter draws our attention to important questions of how mathematics educators can prepare teachers to teach critically and mathematically in an intertwined manner. The authors offer a set of principles based on critical mathematics education, as well as the theory of post-normal science, that are designed to inform classroom practice. Post-normal science reconceptualises the relationship between science, policy and democracy in the context of global challenges featuring high levels of conflict, uncertainty, and urgent risks. Barwell and Hauge prompt us to consider what a mathematics pedagogy for the climate might look like, a question we could purposefully ask in each of our specific contexts. They present three groups of principles relating to forms of authenticity, forms of participation, and reflecting on and with mathematics. These principles are a starting point for thinking and teaching about mathematics teaching in relation to climate change specifically, but are also helpful for planning critical mathematics tasks, problems and teachings in relation to other (post-normal) topics.

From Bergen, following in the tradition of Stieg Mellin Olsen, Lisa Steffensen, Rune Herheim and Toril Eskeland Rangnes write about bringing climate change into the mathematics classroom. Their starting point is the idea

that students' and citizens' understanding of climate change depends on deep critical mathematical understandings, such as knowledge about how mathematical models work to predict future climate changes. With this in mind, they initiated a research project with three lower secondary school teachers and analysed the teachers' choices and arguments for when they included climate change examples in their mathematics classes. In particular, the research focused on "identifying the potential for facilitating students' awareness and understanding of the formatting power of mathematics." They show how these three teachers emphasise the importance of being critical and mathematically literate with the purpose of both understanding and influencing society in their classroom teachings. They close by highlighting the central role teachers have in this way of teaching. They also reveal how they needed to develop further awareness both about mathematical argumentation and "of how they themselves make use of the formatting power of mathematics in their teaching and facilitation for critical learning."

The final chapter in this part, by Jennifer Hall and Richard Barwell, presents a critical mathematics education analysis of the mathematical formatting of the body mass index (BMI) and the societal power that it has. They examine the history and recent use of this measure to show mathematics affects our view of weight, obesity, and body image. They argue that mathematics contributes "to the creation of obesity as a concept and as a problem, and thus shapes people's experiences of obesity and of themselves." They systematically guide us through the history of the BMI formula and show examples of its construction of "obesity discourses" and how these are used in, for example, news media, schools, and insurance companies. In this chapter, a critical mathematical analysis is used to draw attention to the way that the discursive power of mathematics transforms the rich diversity of human life—in this case, human bodies—into "a normative, prescriptive, and ideological abstraction." This transformation then supports forms of intervention and control, and problematic normative discourses of body image, gender, and race.

The four chapters in this part illustrate different ways of applying critical mathematics education to urgent issues in order to promote democratic citizenship and political engagement. These chapters extend previous work in critical mathematics education, for example by focusing on environmental and health issues. They illustrate how democratic citizenship can be fostered in mathematics classrooms by exploring landscapes of investigation with schoolchildren and their teachers. Democratic citizenship also needs detailed analyses of particular mathematical interventions in society, with particular attention to their historical dimension. As Foucault demonstrated, to understand the contemporary organisation of knowing, it helps to trace a historical

trajectory. This kind of critical mathematics analysis offers great potential for researchers to challenge public discourses around phenomena like obesity and climate change, and other post-normal challenges.

3.4 *Epilogues*

The book ends with three Epilogues, with the purpose of inviting scholars with a range of voices and experiences to reflect on applying critical mathematics education, as presented in the preceding chapters. The first is by Ulrika Ryan and Lisa Steffensen, contributors to some of the chapters in the book, who represent up-and-coming voices in this field. The second is by Anita Rampal, whose critical reflections, deeply embedded in the Indian context, with social justice issues like poverty, inequity and caste, challenge our thinking about applying critical mathematics education in contexts very different from Euro-centric ways of thinking and knowing. The final epilogue is by Swapna Mukhopadhyay and Brian Greer, both of whom would, in Swedish metaphor and with great respect, be called “old foxes” of critical mathematics education research. All three epilogues are extended reflections on implicit and explicit themes that appear in the main chapters of the book and prompt new directions for the future. The three epilogues draw out the contemporary relevance of the contributions to this book, clearly influenced by the events of a particularly tumultuous 2020.

4 **Concluding Remarks**

Greta Thunberg is right. She and her friends should not have needed to be present at the United Nations Climate Action Summit in New York City in 2019, demonstrating and taking action for climate change. She should have been in school, as should all her demonstrating friends and young activists. The irony is that Greta Thunberg and all the other activists probably learned a great deal more about applied mathematics, science, social studies, and so on, and about the critical use of mathematics, when demonstrating in city centres and talking at meetings of world leaders, than if they had been back in school, solving simultaneous equations in their regular mathematics classes.

Does this really have to be the case? What is our role as teachers of mathematics, or educators of teachers of mathematics? Should we address this dimension of mathematics in our work? We believe that we should. If anything, all the chapters in this book have taught us the need for a more societally important, politically relevant, democratic mathematics teaching at all levels of the education system. Climate change is an urgent global topic, as is

the aftermath of the COVID-19 pandemic in which we find ourselves. Invisible women in technological development and the impact of big data where the use of mathematics contributes to inequality or social injustice are other examples of the global use of mathematics that impact us as individuals and in our personal lives. However, applying critical mathematics education is equally important in local contexts. Examples like drinking water challenges in Indigenous communities, BMI-measures that affect healthcare decisions, a young boy in a rural school questioning if mathematics is good for society, post-colonial documents with powerful numbers, or the complexity of teaching post-apartheid university mathematics courses in South Africa are all local contexts where applying critical mathematics education can make a difference.

In this book, the contributors show how some version of critical mathematics education can be applied in many different ways and in a wide variety of contexts to offer alternative ways of thinking about mathematics in schools, in universities, in research and in society. Critical mathematics education can be applied to projects of decolonisation, of ethnomathematics, and empowerment. It can be applied to questions of race, racism, obesity or climate change. It can be applied in classrooms or in communities. It can be applied as theory, as a form of analysis or as a way to think about pedagogy. Critical mathematics education has developed many principles and concepts. How else can they be applied?

We find ourselves finishing this introduction during a peak of the COVID-19 pandemic. Thunberg's *cri de coeur* about not being in school, is now, more than a year later, the reality for many children and youth in the world during the widespread lockdowns. The pandemic is a time when we see mathematical models, diagrams and graphs from all over the world, perhaps more than ever before. We notice them every day, in all kinds of media. As noted by a friend on Annica's Facebook wall, the number of exponential graphs on Facebook is increasing exponentially. To the questions posed above, we now also ask how this COVID-mathematics will influence our future teaching or our students' interest in societal mathematics, when schools and universities open again.

Our hope is that the tremendous value of applying critical mathematics education, in all the different ways shown in this book, will open up the range of possibilities for research and practice in mathematics education in order to focus on local and global challenges in local contexts. It has been a privilege to work with the texts for this volume. We hope that this work will inspire new creativity in mathematics learning, teaching and curriculum, so that all children can become as critically mathematically literate as Greta Thunberg and all her activist friends around the world. We should not fail them in any mathematics classroom.

Notes

- 1 <https://news.un.org/en/story/2019/09/1047052>
- 2 <https://www.theguardian.com/world/2020/apr/24/sexism-on-the-covid-19-frontline-ppe-is-made-for-a-6ft-3in-rugby-player>
- 3 Indian Residential Schools were state-sponsored schools to which Indigenous children were forcibly sent from the nineteenth century onwards, with the explicit goal of erasing their Indigenous cultures and languages. Children were sent hundreds of kilometres away from home from the age of five. The last such school closed in 1996.
- 4 This observation raises the uncomfortable possibility that the loose coalition of researchers we identify as critical mathematics educators needs to question its own biases and that we need to consider the biases in our characterisation of this coalition.

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