IV

URBAN ELITES AND CIVIC LIFE

ROMAN ACTUARIAL SCIENCE
AND ULPIAN'S LIFE EXPECTANCY TABLE

By
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1. Introduction

Roman lawyers must have been well aware of the problem of contingency. A simple case may suffice to illustrate the phenomenon. I have contracted with someone to pay me a certain amount of money in the event of the bankruptcy of one of my debtors (*fideiussio indemnitis*).¹ This is a simple form of credit insurance and it is a wager. The event may or may not happen; if it happens, it may happen within a short time after I have contracted with the guarantor, or it may happen a long time after. Roman 'bankers' (*argentarii*) were specialised in this kind of credit insurance (*receptum argentarii*)² and must have been able to assess their chances in order to fix a competitive price for their services.³ Today the problem of assessing chances is, of course, paramount with life-insurances and it is generally believed among lawyers and legal historians that contracts like these only proliferated after the development of modern actuarial sciences, that is after Johann de Witt (1625-1672) in the Netherlands and John Graunt (1620-1674) in England had shown the way to compose sufficiently accurate life expectancy tables.⁴ Be this as it may, Roman lawyers must have had considerable actuarial knowledge as well, if only to estimate the capital value of annuities, sustenances and other life interests. It is the context of Aemilius Macer's rule on life expectancy and what is generally referred to as 'Ulpian's life expectancy table'. It has been contended that Ulpian's table was not

¹ See, for example, *Dig.* 50.16.150 (Gaius): *Si ita a te stipulatus fuero: 'quanto minus a Titio consecutus fuero, tantum dare spondes?'*, non solet dubitari, quin, si nihil a Titio fuero consecutus, totum debeas quod Titius debuerit. For more details see Papinian in *Dig.* 45.1.116.

² On *receptum argentarii* see Jean Andreau, *La vie financière dans le monde Romain. Les métiers de manieurs d'argent* (Rome 1987), 597 ff.

³ See *Dig.* 17.1.6.7, where a certain Maurus Paulus offered his services as guarantor in return for a price. On this text G. MacCormack, 'Periculum', *Zeitschrift der Savignystiftung für Rechtsgeschichte, Rom. Abt.* 109 (1979).

⁴ See, for example, H. Coing, *Europäisches Privatrecht II* (München 1989), 563.

empirically based: ‘how could it be?’⁵ In this paper we will consider the possible empirical basis of Ulpian’s table and the way a fairly accurate life expectancy table could be computed on that basis.

2. Ulpian’s table

Little is known about the Roman lawyer Aemilius Macer. It has been suggested that he lived under the emperors Caracalla and Alexander Severus. If so, he must have been a younger contemporary of Ulpian.⁶ It may be surmised from what has been handed down to us from his writings,⁷ that he had a certain propensity for public law. He wrote a treatise on the Roman inheritance tax, the *lex Iulia de vicesima hereditatium*. It is in one of the fragments from this book, surviving in Justinian’s Digest, that Macer gives an account of the way the capitalised value of a life interest could be established for taxation purposes.

Dig. 35.2.68 pr. (Macer, libro secundo ad legem vicesimam hereditatium) Computationi in alimentis faciendae hanc formam esse Ulpianus scribit, ut a prima aetate usque ad annum vicesimum quantitas alimentorum triginta annorum computetur eiusque quantitatis Falcidia praestetur, ab annis vero viginti usque ad annum vicesimum quintum annorum viginti octo, ab annis viginti quinque usque ad annos triginta annorum viginti quinque, ab annis triginta usque ad annos triginta quinque annorum viginti duo, ab annis triginta quinque usque ad annos quadraginta annorum viginti. ab annis quadraginta usque ad annos quinquaginta tot annorum computatio fit, quot aetati eius ad annum sexagesimum deerit remisso uno anno: ab anno vero quinquagesimo usque ad annum quinquagesimum quintum annorum novem, ab annis quinquaginta quinque usque ad annum sexagesimum annorum septem, ab annis sexaginta, cuiuscumque aetatis sit, annorum quinque. eoque nos iure uti Ulpianus ait et circa computationem usus fructus faciendam. solitum est tamen a prima aetate usque ad annum trigesimum computationem annorum triginta fieri, ab annis vero triginta

⁵ K. Hopkins, ‘On the probable age structure of the Roman population’, *Population Studies* 20 (1966-1967), 264 n. 22: ‘Ulpian’s table is <not> empirically based – how could it be?’. But see K.J. Beloch, *Die Bevölkerung der griechisch-römischen Welt* (Leipzig 1886), 44: ‘Ulpian’s Zahlen <sind> offenbar auf rein empirischem Wege gefunden, und zwar in recht roher Weise’.

⁶ W. Kunkel, *Herkunft und soziale Stellung der römischen Juristen* (Graz-Vienna-Cologne 1967), 256.

⁷ See O. Lenel, *Palingenesia Iuris Civilis* I (Leipzig 1889), 561-574.

tot annorum computationem inire, quot ad annum sexagesimum deesse videntur. numquam ergo amplius quam triginta annorum computatio initur.

‘Ulpianus says that this is the formula to be adopted in making the estimate of sustenances to be furnished. The amount bequeathed to anyone for this purpose from the first to the twentieth year is computed to last for thirty years, and the Falcidian portion of that sum shall be reserved. From twenty to twenty-five years, the amount is calculated for twenty-eight years. From twenty-five to thirty years, the amount is calculated for twenty-five years; from thirty to thirty-five years, the amount is calculated for twenty-two years; from thirty-five years to forty years, it is computed for twenty years; from forty to fifty years, the computation is made for as many years as a person lacks of the sixtieth year after having omitted one year; from the fiftieth to the fifty-fifth, the amount is calculated for nine years; from the fifty-fifth to the sixtieth year, it is calculated for seven years; and for any age above sixty, no matter what it may be, the computation is made for five years. Ulpianus also says that we use the same rule in making the calculation with reference to the legacy of an usufruct. Nevertheless, it is the practice for the computation to be made for thirty years from the first to the thirtieth, but after the age of thirty years it is made for as many years as the legatee lacks of being sixty; hence the computation is never made for a longer time than thirty years’.

The compilers of Justinian’s Digest placed this extract from Macers book in the second section of the thirty-fifth book of the Digest on the *lex Falcidia*, a *plebiscitum* from 40 BC, fixing the portion of his estate a testator was free to dispose of in his last will at three-quarters. The so-called *quarta Falcidia* was to be set-aside for the heirs.⁸ In estimating the *quarta Falcidia*, it was necessary to establish the value of legacies and consequently the question arose how the value of life interests, such as the legacy of an usufruct or a life-long sustenance, was to be estimated, hence the need for a calculated assessment of human life expectancy.⁹

⁸ On the *quarta Falcidia* see M. Kaser, *Das römische Privatrecht* I (München 1971), 756.

⁹ It is rather odd to find a reference to the *lex Falcidia* in an extract from a book on death duties. The reference to the *lex Falcidia* is almost certainly to be attributed to Justinian’s compilers. Originally, the fragment dealt with problems concerning the computation of the value

The fragment contains two computation-methods for calculating the capital value of life interests: one reported by Ulpian¹⁰ and another that is represented by Macer as the normal method for estimating the value of these interests. It is generally believed that 'Ulpian's' table was meant as a refinement of the usual computation-method as reported by Macer.¹¹

3. The empirical basis of Ulpian's table

One of the problems with Ulpian's table in the study of Roman demography concerns the accuracy of its calculations in view of the epigraphical material available to us *now*. The results of the table do not seem to reflect the present empirical data. Of course, we are aware of Keith Hopkins' inspiring article on the relative value of epigraphical material in making demographical assessments on Roman society in general,¹² but some remarks on the reliability of epigraphical data have to be made, as doubts have even been casted on their general accuracy as well. Historians involved in this debate generally overlook some important facts of Roman law. Take, for example, Bruce Frier's casual remark on 'the well-known propensity of Romans to be uncertain about their

of legacies of life interests in relation to the levy of the *vicesima*. This death duty was obsolete in the time of Justinian (see *CJ* 6.33.3 pr.). However, the compilers of the Digest rightly felt that the problems involved were the same as with the computation of the *quarta Falcidia* (cf. R. Duncan-Jones, *Structure and scale in the Roman economy*, Cambridge 1990, 96 n. 10). A conviction shared, no doubt, by the contemporaries of Macer himself. We owe the survival of Macer's fragment (and Ulpian's life expectancy table) to this coincidence; see P. Stein, 'Generations, life-spans and usufructs', *Revue Internationale des Droits de l'Antiquité* 9 (1962), 340 ff.

¹⁰ We do not know whether it is to Ulpian himself, to whom the life expectancy table reported by him should be attributed. Macer merely says that 'Ulpian writes that this is the formula' (*hanc formam <computationis> esse Ulpianus scribit*). It is equally uncertain to which of Ulpian's writings Macer refers. O. Lenel (*Palingenesia Iuris Civilis* II (Leipzig 1889), 1198 n. 2) suggested a section of Ulpian's book *De omnibus tribunalibus* dealing with legacies of life sustenances. A surviving fragment (*Dig.* 2.15.8.10) does indeed mention the estimation of such interests and may originally well have contained the table.

¹¹ Stein 1962, op. cit., 343 and B. Frier, 'Roman life expectancy: Ulpian's evidence', in *Harvard Studies in Classical Philology* 86 (1982), 213 ff; 219 and 224 ff.

¹² K. Hopkins, 'Graveyards for historians', in Fr. Hinard, ed., *La mort, les morts et l'au-delà dans le monde romain* (Caen 1987), 113-126.

ages'.¹³ Now there is a well-known propensity of some modern people – actors, opera-singers and society-belles among them – to be uncertain about their age, but this does not mean that we are in general uncertain about our age and there was no such uncertainty among the Romans. To the contrary: they *had* to be certain about their age. The very existence of a table like Ulpian's testifies to it and the law required it. All contracts entered into by minors were null and void¹⁴ and after the *lex Laetoria* (circa 200 BC) all contracts entered into by persons under the age of twenty five could be set aside by the court.¹⁵ Justinian's Digest contains a whole section on the many problems involved here¹⁶, none of them caused by difficulties of age-assessment. The law as set out there presupposes an easily certifiable way of assessing the age of the persons involved beyond any doubt. The *lex Aelia Sentia* – to name but one other example – provided for the nullity of informal emancipations by a slave-owner younger than thirty.¹⁷

There are no traces in Roman legal literature of specific problems caused by 'the well-known propensity of the Romans to be uncertain about their ages'. They were not and the main reason they were not is the fact that their age was a matter of record. Since the days of the emperor Marcus Aurelius, all births had to be recorded in a public birth register within thirty days after they had occurred. In Rome, the *praefectus aerarii* kept public records of births; in the provinces they were kept by a *tabularius publicus* working in the governor's residence.¹⁸ In case of a dispute, authenticated extracts from these records were

¹³ Frier 1982, op. cit. (n. 11), 226.

¹⁴ Gaius 3.109. There was a controversy over the age at which a minor became a *puber* and consequently able to conclude a binding contract. The 'Sabinian' school contended that it depended on the individual; the 'Proculian' school, however, held that girls reached *pubertas* after 12 years and boys after 14 (see Gaius 1.196). The latter contention, confirmed by Justinian (*CJ* 5.60.3), is untenable without easily certifiable ages.

¹⁵ For details see Kaser 1971, op. cit. (n. 8), 276-277.

¹⁶ *Dig.* 4.4 (*De minoribus viginti quinque annis*).

¹⁷ Gaius 1.18 ff. Another compelling reason for a Roman to be certain about his age was, of course, taxation. Ulpian says that a citizen has to mention his age in his tax declaration (*Dig.* 50.15.3: *aetatem in censendo significare necesse est*), if only because in some areas, as for example Syria, persons over the age of sixty-five and under the age of twelve (females) or fourteen (males) were not liable for certain taxes.

¹⁸ On these records see *SHA, Marcus Antoninus* 9.7-9. Before Marcus registration of births was required by custom, which explains the presence of extracts from the public record before his reign (on this point: H.J. Scheltema, 'Professio liberorum natorum', in *Tijdschrift voor*

available on request.¹⁹ Some of them have survived and it should be noted that there are specific references in them to the provisions of the *lex Aelia Sentia* and similar provisions in the *lex Iulia Poppaea*.²⁰ This shows that a Roman who had to know his age could easily establish it from a public record office.

It is true that Roman law did not provide for a genuine *population* register, as it seems that *deaths* were not officially recorded.²¹ They seem, however, to have been unofficially recorded as a matter of course in the *rationes Libitinae*, the records of deaths kept by the priests of the temple of (Venus) Libitina.²² We do not know whether the data contained in these records were available to the public at large. There is no indication that they contained a reference to the data of *birth* of the deceased. There was, however, an alternative and very public record of mortalities available, the Roman cemeteries and *columbaria*, containing gravestones and *ollae* with – even to modern standards – unusually accurate information on the ages of the deceased. Death was ubiquitous all around Rome and the other cities of antiquity and the abundance of funeral inscriptions offered ample material for the composition of a fairly accurate life expectancy table. This raises another important question.

Some emphasis has recently been laid on the fact that a relatively high percentage of the epitaphs available to us now are from the tombs of freedmen.²³ It may well be that they outnumbered the *ingenui* even in antiquity, which raises the question of whether a freedman (or – which amounts to the same – a slave) was able to assess his age with as much certainty as a freeborn Roman could.

Rechtsgeschiedenis 14 (1936) 86 ff). See also F. Schulz, 'Roman registers of birth and birth certificates', in *Journal of Roman Studies* 32 (1942), 78 ff. and 33 (1943) 55 ff; L. Wenger, *Die Quellen des römischen Rechts* (Vienna 1953), 812-813 and Kaser 1971, op. cit. (n. 8), 273.

¹⁹ An important fact overlooked by most writers on the subject, such as A.R. Burn, 'Hic breve vivitur: A study in the expectation of life in the Roman Empire', in *Past and Present* 4 (1953), 4 ('in the absence of any registration of births') and T.G. Parkin, *Demography and Roman Society* (Baltimore-London 1992), 37.

²⁰ *Fontes Iuris Romani Anteiusianiani* III (Florence 1968), nos 1, 2 and 3.

²¹ Parkin 1992, op. cit. (n. 19), 36.

²² On this record see Dionysius of Halicarnassus, *Antiquitates Romanae* 4.15, citing Lucius Piso who attributed the institution to Servius Tullius. On the *ratio Libitinae* see Suetonius, *Nero* 39. See for further information J. Marquardt, *Das Privatleben der Römer* I (Leipzig 1886), 385; G. Wissowa, 'Libitina', in W.H. Roscher, *Ausführliches Lexikon der Griechischen und Römischen Mythologie* (Leipzig 1890-1897) II, 2034.

²³ Duncan Jones 1990, op. cit. (n. 9), 85, referring to research by Solin, Taylor en Weaver.

There seems to be little doubt that he could. We know from inscriptions on the graves of slaves that they could indicate their age accurately, a rather remarkable phenomenon in itself that is rarely emphasized. A slave-holding society is seldom concerned with the age of slaves, as one is rarely interested in the precise age of the chattels constituting a household. We know, for example, that slaves in the former slave-holding states of the United States of America were unaware of their age. One of them, who wrote his autobiography, explicitly states that 'I do not remember to have ever met a slave who could tell of his birthday'.²⁴ Roman slaves could and, again, it is the law that made it a necessity. It is explicitly stated in the *lex Aelia Sentia* that slaves younger than thirty years of age could only be emancipated provided that certain formalities were complied with.²⁵ Information on his exact age was therefore crucial to a freedman. True as it may be that Roman law did not provide for a mandatory registration of births of slaves generally²⁶, a private record was kept by their owners, as modern Dutch farmers still do with their pedigree cattle. There are references to these records (*libelli familiae*) in the Digest.²⁷ Furthermore, the Codex Justinianus reports an interesting case concerning a dispute over the property of a slave where the plaintiff was unable to prove by his private records that the slave was born to him as a *verna*.²⁸ The case seems to presuppose that such records were kept as a matter of course.²⁹ It seems, therefore, rather certain that Romans – freemen, freedmen and even slaves – were not only well

²⁴ From *Narrative of the life of Frederick Douglass, an American slave, written by himself* (Garden City, NY, 1963).

²⁵ Gaius 1.18 en 20.

²⁶ E. Herrmann-Otto, *Ex ancilla natus: Untersuchungen zu den 'Hausgeborenen' Sklaven und Sklavinnen im Westen des römischen Kaiserreiches* (Stuttgart 1994), 234. Egypt was an exception: see R.S. Bagnall and B.W. Frier, *The demography of Roman Egypt* (Cambridge 1994).

²⁷ Dig. 32.99 pr. (Paul). The case concerns a legacy of some *servi urbani*, slaves not employed in farm labour. Paul states that the members of the *familia* belonging to this category can be derived *ex libellis familiae*. He further mentions the *cibarii*, the records of food supply to the slaves. A Roman household of some size had a *cellarius*, who was employed 'to keep the records in order' (*ut rationes salvae sint*): Dig. 33.7.12.9 (Ulpian).

²⁸ CJ 3.32.10.

²⁹ The 'procurators' administering the estates belonging to the imperial domain must have kept similar records: Herrmann-Otto 1994, op. cit. (n. 26), 124 n. 71.

informed about their age, but that the ages recorded on their graves were as a rule correct. It may be that this information was less precise in the provinces, which may account for some irregularities there, such as the ‘age-rounding’ phenomenon³⁰, but Ulpian’s table was only concerned with Rome.³¹

4. Ulpian’s table and its purpose

Too much emphasis is usually laid on the fact that Ulpian’s table survives within the context of the levy of the *vicesima hereditatium*, the Roman inheritance tax. It obscures the fact that the establishment of some way of assessing life expectancies was required in the normal private practice of the law as well. In order to appreciate this assessment, two facts concerning the socio-economic background of Roman private law in imperial times have to be emphasised. The first concerns the fact that Roman commercial legal practice was mainly concerned with litigation over inheritances and legacies, much like modern commercial legal practice is mainly about corporation law. It was there that large fortunes were disputed and large fortunes were to be gained by council. The second fact that needs to be emphasised is related to this. It was quite common for heirs and legatees *not* to take possession of estates and property bequeathed to them, but to sell them to the highest bidder. It was an easy way to capitalise on the advantages gained without carrying the burdens. The Codex Justinianus, as well as the Digest, contain large sections on the ‘sale of inherited estates’.³² There was a genuine ‘estate’-market in the Roman economy, much like the modern ‘mergers and acquisitions’ business. The sharp speculators involved in it had to be able to assess the value of an estate in a lump sum. In order to do so, they had to take into account all the provisions of a will, especially the rather common legacies of usufructs, annuities, sustenances and other life interests.

³⁰ On this phenomenon: R.P. Duncan-Jones, ‘Age-rounding, illiteracy and social differentiation in the Roman Empire’, *Chiron* 7 (1977). It seems to be frequent in those sources only, which represent someone’s guess at another person’s age: W. Scheidel, *Measuring sex, age and death in the Roman empire* (Ann Arbor 1996), 84 ff.

³¹ Slaves were entitled to join funerary societies (*collegia tenuiorum*). By a lucky coincidence the articles of association of one of them survive (ILS 7212). They explicitly provide for the membership of slaves. It may well have been that these societies kept records of the deaths of their members, as Stein 1962, op. cit. (n. 9), 343, thought, but no traces of them have survived; see Parkin 1992, op. cit. (n. 19), 35.

³² Dig. 18.4 and CJ 4.39 (*De hereditate vel actione vendita*).

After taking over the inheritance, these burdens rested on the buyers and consequently their value had to be assessed in order to fix a price. There can be little doubt that rules of thumb were observed among these traders regarding the life expectancies of the legatees of such allowances at a given age. One that comes to mind is the rather crude rule that Aemilius Macer gives as an alternative to Ulpian's table. This raises the question why and by whom a more refined method was developed and, of course, how it could have been done.

Ulpian had been a successful practising lawyer, before entering into his 'procuratorial' career.³³ He must have been acquainted with the methods employed by estate-speculators in assessing a life expectancy. Even without entering into the problems concerned with this trade, he must have been well aware of their rules of thumb, as they had to be employed within another context as well. In assessing the statutory share of an inheritance that had to be reserved for the heir, fixed by the *lex Falcidia* at a quarter of the capitalised value of an estate, the value of all legacies had to be accounted for, including those providing for an annuity, an usufruct, or other life interests.³⁴ In assessing the value of such legacies, an older jurist, Marcellus, refers to 'the price for which these legacies are sold on the market'.³⁵ This is a direct reference to the Roman estate market and implicitly to the rules of thumb employed there. Ulpian knew all this when he entered the imperial bureaucracy. We know he was a *procurator a libellis* at some point in his career, in which capacity he had to prepare the imperial *decreta* concerning appeals against, for example, the decisions taken by local procurators of the inheritance tax (*procuratores XX hereditatium*). Some of these appeals may well have originated in disputes over the way an annuity was valued and it was on occasions like these that a rule had to be found by the imperial court. The small circle of highly trained legal experts mentioned in Julius Paulus' 'reports' of the imperial *decreta* of the time – Papinian, Ulpian, Messius and, last but not least, Paul himself – may have found the crude rule too arbitrary. All of them were familiar with the problem involved and it is consistent with the highly rational approach to the law that is such a striking

³³ On Ulpian's career see T. Honoré, *Ulpian* (Oxford 1982), 15 ff.

³⁴ On this see Ulpian in *Dig.* 2.15.8.10.

³⁵ *Dig.* 35.2.55: *cum Titio in annos singulos dena legata sunt et iudex legis Falcidia rationem inter heredem et alios legatarios habeat, vivo quidem Titio tanti litem aestimare debeat, quanti venire id legatum potest, in incerto posito, quamdiu victurus sit Titius*

characteristic of Roman law at this stage of its development that some effort was made to improve on the usual method other than by mere guesswork. But could it be done? What would a rational man have done in the time of Ulpian if he were asked to make an alternative to the crude rules of thumb as employed by the estate-traders?

5. A rational and empirical approach to Ulpian's table

Most, if not all, modern literature on Ulpian's table approaches it on the basis of the empirical material available *in modern times*, i.e. the surviving epigraphical material. It is on the basis of a confrontation with that material that the reality of the table is evaluated. We are, however, not in a position to do so, as the material available to us *now* is hardly sufficient for a fair evaluation of the quality of Ulpian's table. It is questionable, to say the least, whether the surviving epigraphical material on tombstones is a representative sample of the tombstones present in antiquity at various times and in various regions. It is another question whether the material available *in antiquity* could have been treated as a representative sample at the time. Ulpian and his contemporaries did not have the privilege of sharing our knowledge of Keith Hopkins's warning against the use of epigraphical material for demographical purposes and may therefore well have done what any sensible man would do: consult the only public register available, the graveyards.³⁶

The compiler or compilers of Ulpian's table were not interested in a survey for demographical purposes, but in solving a practical problem in as practical a way as possible. In order to understand the ingenious way the table has been composed, some basic statistical concepts have to be explained.

Median life expectancy

In modern insurance business actuaries compute the life expectancy of an individual on the basis of the *median* life expectancy of the cohort he belongs to.

³⁶ As will be shown later, the compiler of the table was well aware of the fact that the graves of children under the age of 10 were underrepresented. He may even have been aware of the fact that age-reporting on tombstones was not universal and seems to have been 'an essentially middle-class and lower-middle-class institution': Burn 1953, op. cit. (n. 19), 7. If so, it must have suited him well, because Roman private law was a very middle-class institution.

The concept of median life expectancy is important.³⁷ Consider the following example.

Suppose someone belongs to a cohort of 1000 persons of 15 years of age. Suppose also that, when the same cohort is 45 years of age, 500 of the individuals belonging to it (50%) have died. Then the age of 45 is the 'median' age of the cohort. It is the age at which half of the cohort is still alive.

Assessing the median age³⁸ must have been important to Roman estate-speculators, the businessmen buying estates burdened with legacies of life interests. Suppose, for example, an inheritance is offered to him, containing a legacy of an annuity to someone 15 years of age at the time it becomes due and the estate is offered to a speculator. If the median age of the cohort the legatee belongs to is 45, the buyer knows he may expect to pay for thirty years. After discounting this liability from the estimated value of the assets of the estate, he is capable to fix the price he is willing to pay. If the legatee dies before he has reached the age of 45, the buyer has won in the bargain; if the legatee dies after that, he has lost. So, his expected loss equals his expected profit.

In modern societies such cohorts are easy to follow, but in antiquity they were not.

Establishing a median value

Fig. 1 is an example of the determination of a median value. It shows the development of the cohort referred to in the example mentioned above, the one of persons of 15 years of age.

³⁷ Burn, op. cit., 31; Frier 1982, op. cit. (n. 11), 220.

³⁸ The concept of 'median age' should not be confused with 'average' age. The latter is a statistical concept of the last two or three centuries.

Fig.1
Age and number of survivors

We start with 1.000.000 people, a large number, but that is the way actuaries work. Five years later, 900.000 are still alive. We then look for the 50% threshold, which appears to be 45 years. So in this example (derived from Bruce Frier's article on Ulpian's table³⁹), the median expected life of the cohort is 45 years. Now the median life expectancy of this cohort of 45 years can be determined. That will be reached when half of them have died, which is at the age of sixty. Of course, this holds true only in a situation of linear development, but, as Bruce Frier has indicated, that seems to have been the case in Roman society. We will not challenge that contention and stay with it, just for argument's sake. By way of comparison a second graph has been added in Fig. 1, referring to the development of a similar cohort in modern times. It is obvious that a dramatic change has occurred since antiquity. Due to the so called 'morbidity compression', the downfall occurs at the end, whereas in Roman times the curve seems to have had a neat and almost linear shape.

We can now assess the median expected length of life for each age group. For convenience's sake, only the years still to live were plotted.

³⁹ Frier's data will be used for illustrative purposes.

Fig. 2
Age and expected years to live

Two essential conditions have to be met in order to compose a table like this:

- (a) The population register must contain dates on birth *and* death and
- (b) These data must be easily accessible. Although there was indeed a public record of births in Ulpian's time, we do not know whether the officers of the temple of Libitina (the unofficial death registration) recorded dates of death *and* birth and we do not know whether these records were as easily accessible as the public birth register.

It is fair to surmise that the conditions to be met in order to construct a table like Ulpian's were not fully met in his time. But was it possible to construct a proxy on the basis of empirical data – epigraphical material – available to Ulpian in his own time?

Composing Ulpian's table

True as it may be, that the Romans were poor at axiomatic mathematical reasoning, they were highly gifted in applied mathematics, as Vitruvius' work

and the writings of the *agrimensores* (the Roman land-surveyors) testify. In order to construct a table like Ulpian’s two conditions must have been met at the time:

- (a) A stable demography over a certain period
- (b) An (almost) linear curve of survivors for each age cohort⁴⁰

If these conditions were met in Ulpian’s time, it can be shown that he or his contemporaries must have been able to compose the table just by counting the new graves on one random graveyard and noting the ages of the deceased as reported on most of them. A linear development is taken for granted, as there seems to be little doubt among historians that – except, of course, under extreme conditions like the plague – such a situation generally prevailed in Rome in the age of the Antonines.⁴¹ It may, moreover (and even more importantly), well have been a situation taken for granted by Ulpian and his contemporaries.

Fig. 3

Age and number of survivors: almost linear

⁴⁰ The linear curve is not an absolute condition, but simplifies the observations.

⁴¹ See, for example, Frier 1982, op. cit. (n. 11), 241.

The number of survivors in an age group is the key to Ulpian's life expectancy table. Before going into the – very simple – mathematics of the table, one important presumption will have to be stressed. The demographic morphology – the rate of births and deaths – is supposed to have been stable at the time the table was composed. It is an important issue, because – as will be shown later – it renders superfluous the cumbersome tracking of entire age-cohorts over the years. Now a simple mathematical reasoning follows.

First we follow our cohort. It is indicated by the letter C. Suppose all the members of our cohort have been born in the year 200 AD. The number of persons living in the year 215 AD is represented by $L(15)$. Five year later this number is – obviously – less. It is indicated by the expression $L(20)$. If all the members of our cohort, deceased in the last five years, have graves, the number of graves is indicated by $G(20)$. This number equals the formula $L(15)-L(20)$. A mathematician renders this in a general formula (1):

$$L(n) = L(n-5) - G(n),$$

where $n = 20, 25, \dots$ years

For most non-mathematicians this may look somewhat complicated, so a less technical description is called for. Suppose we know the number of the cohort C in 215 AD. We then count the graves of the deceased of this cohort in 220, i.e. all persons of 15-20 years who died in the period 215-220. The number of survivors in the cohort in 220 AD is then simply the number of persons living in 215 minus those who died in the previous five years. Over time, we can repeat this procedure every five-year to determine the number of survivors. Suppose the whole cohort has died out in 285. Then obviously we stop the procedure and construct a table of survivors as shown earlier. This leads to median ages, which is necessary for the constructing of a table like Ulpian's.

For obvious reasons, this *cannot* have been the way Ulpian's table was constructed, if only because it was a very unpractical procedure indeed. Consider the following problems:

- (a) The number of persons in the cohort in 215 is not known. This is a corollary of the high mortality among children – only 30% of all

newborn babies will have reached 10 years of age. Their births may have been registered, but their deaths may not have been and their graves hardly existed. So a starting number of the number of persons in a cohort may be given, but it is hard to follow in the early years.

- (b) The length of the procedure is unpractical. Following a cohort during 70 years is simply too much. This problem will be dealt with after the problem of the absence of a starting number has been tackled.

As has been observed, we do not know the starting number. But we do know the ending number: in 285 AD this number equals zero.⁴² So, why not count *backwards*?

Counting backwards

Suppose our cohort has become extinct in 285 AD. That means that no members are present in that year and $L(85)$ equals zero. Now we can rewrite formula (1) as follows:

$$L(n-5) = L(n) + G(n)$$

Substituting $n=85$ gives

$$L(80) = L(85) + G(85)$$

Since $L(85) = 0$, we have

$$L(80) = G(85)$$

By using this formula, we should be able to compute $L(75)$, since $G(80)$ and $L(80)$ are known. Remember: all numbers of the graves of our cohort are known (since the age of the members of each particular cohort is indicated on the tombstones) and are counted each five years. By repeating this procedure, we are able to compute all the values of L recursively. This may look rather frustrating to a non-mathematician, but it is a simple procedure and can be less formally explained. We are simply counting backwards. We first count the new graves of the cohort in 285 AD. So we get the number of the living in 280. But if we know the living in 280, we can easily get the number of the living persons in 275 AD, by simply adding the new graves of the deceased persons in the

⁴² This number is postulated for practical reasons. Few persons belonging to the cohort will have lived longer than 85 years.

cohort in the period 275-280 A.D to the number of living in 280 AD. And so on. Proceeding this way, we end up by all the living in 215 AD.

A short observation period

It is obvious that the simple method as outlined above would still take an observation period of 70 years. Consequently, it is impractical and – even more important – rather superfluous. It is not necessary to follow an entire cohort from the cradle to the grave. As has been emphasized before, we assume a stable demography, meaning that cohorts of a given age in a given year are the same overtime. So a cohort of people born in 200 AD contains in 230 AD the same number of members as, for example, a cohort born in 240 AD in 270 AD. Consequently, when we count the number of the last cohort in 270 AD, we know all cohorts of thirty years of age, irrespective of the year of birth of the members of that cohort. We, therefore, only have to count the new graves in a given period and note the age of the cohort they belong to, by noting the age of the person in the new grave. In this way, an entire cohort can be reconstructed recursively on the basis of the deceased from other cohorts. The principle can be easily grasped by looking at Table 1.

In contemplating Table 1, attention should be drawn to the bottom-row. It shows the development of the number of graves of our cohort C in the period 220-285 AD. The cohorts starting in 205, 210 *etc.* are identical, since a constant demography is presupposed, so later cohorts show the same development with respect to the number of new graves. So, when, for example, the period 270-275 AD is taken as an observation period, the development of the entire cohort C until that moment can be constructed by noting the number of new graves of different age groups. This means that we can limit our observations to a very short period, which makes it the practical way to operate. There are, however, some problems to be solved.

Child mortality

Counting graves in order to set up a demography will only hold if all graves of all cohorts can be counted. This is not the case. Child mortality is a disturbing factor indeed, since circa 70% of all newborns did not reach the age of 10. In order to overcome the problem of the underreporting effect of child mortality, we start counting the deaths of persons of 15 years and older.

Table 1

COHORT
NUMBER

Very old people

There is a tendency of overreporting the age of very old people. Some areas in the Roman empire are known for a relatively high percentage of centenarians.⁴³ Irrespective of the actual situation in Rome itself, this will not pose a serious problem in a demographic analysis, since the absolute number will be very low and may therefore be neglected.

Not all deceased persons had graves

This can be a serious problem when the analysis is carried out longitudinally. When a cohort is followed throughout its lifetime, then a problem may arise. In this case the burial habits can change gradually, which will disturb the procedure of the analysis of the demography. However, as pointed out earlier, we have transformed the longitudinal analysis into a transversal one, by concentrating on one time interval of five years. This implies that we may assume that those habits will not have changed in that time interval. This does not mean that all deceased persons did have relevant inscriptions on tombstones, but that it was certainly the case for a fixed percentage. This means that we may see the observations of all graves with relevant inscriptions as representative for the (middle class) freeborn men of Rome, if not the population at large.

Changes in population

By transforming the analysis from a longitudinal to a transversal one, we have also made an important assumption about the stability of the demographic morphology. If this morphology is not sufficiently stable, errors may occur. Median values may change by a couple of years if the population growth is of a magnitude of 0,5 % over a long period. According to Parkin⁴⁴ a stable population is likely, a *continuous* growth of 0,5% is not. This means that errors occur, but in a random way, which is normal in this kind of analysis.

⁴³ See the centenarians reported by Pliny from the archives (*vasaria*) of a recent census: *Naturalis Historia* 7.49.162 ff. One wonders whether the ages reported there may not have been bona fide errors, simply caused by confusing the consulates of the date of birth.

⁴⁴ Parkin 1992, op. cit. (n. 19), 85-86; 92-93.

6. The construction of Ulpian's table

As demonstrated above, it is possible to construct a table of expected years to live, given a certain age, by simply observing new graves and noting down the ages of the deceased in a given five-years period. The analysis required in order to arrive at the table is very simple, only a rudimentary arithmetic is needed. The same method can be used if there were registers of deceased Romans recording their ages at the moment of their deaths, as the *rationes Libitinae* may have done. There are however some conditions to be met to make the table a practical one.

The first condition is *simplicity*. A table with too many entries is not practical. It may have been possible to construct a table for the life expectancy of every age group, but memorizing a table like that is a problem. If our assumption is correct and the table was designed for use in the civil service, not only in Rome, but in the entire empire, then it *had* to be concise. A long table containing a double list of more than a hundred numbers (ages and expected years to live) is very susceptible to text-corruption, all the more so because it had to be copied and duplicated by hand. A short and easy to memorise table was therefore preferred. So, most age groups with a five-year's unit will be very practical. There are problems with under- and over reporting, as discussed above. For the young and the old it may not be possible to arrive at representative values of expected years to live. In these cases a choice for a (reasonable) fixed value is logical, if it sufficiently fits the table. Therefore, a simple fixed value for the expected years to live was employed for persons under 20 (30 years) and over 60 (5 years).⁴⁵

Consequently, the table was compressed to 9 age groups with the corresponding expected years to live. However, by doing so discontinuities were introduced. For example a person of 29 years has 25 years to live, a year later his life expectancy is reduced to 22 years. This is an obvious corollary of the limitation to five-year groups. By choosing five-years units, discontinuities are implied if a fixed number of life expectancy years is chosen for the age group.

⁴⁵ The 30 year-period was not chosen randomly: it was commonly believed in antiquity to represent the time-span during which a generation reproduces itself. Censorinus comments on it in his treatise 'On Birthdays' (*De die natali* 17.1). It was also used by Roman lawyers in the law of prescription and in the law of limitations. See *CJ* 7.39.3; cf also E. Levy, *West Roman vulgar law. The law of property* (Philadelphia 1951), 184 ff., and Stein 1962, op. cit. (n. 9), 335 ff

This can be avoided at the cost of a more complicated table. The compiler of Ulpian's table recognized the problem of discontinuity, because he deliberately introduced a continuity correction formula for the age group of 40-50 years as the implied discontinuity within this age-group was too considerable: $60 - \text{age} - 1$. This works very well for the end of the group 35-40 and for the beginning of the group 50-55.

Ulpian's table	
Age	Expected years to live
0 -20	30 years
20-25	28 years
25-30	25 years
30-35	22 years
35-40	20 years
40-50	$60 - \text{age} - 1$
50-55	9 years
55-60	7 years
60 +	5 years

7. Conclusion

We are convinced Ulpian's table was a deliberate attempt at constructing a fairly accurate life expectancy table using simple mathematical methods. It is our contention that, in order to achieve that end, the compiler or compilers of the table *must* have used genuine demographic data (from tombstones, or the *rationes Libitinae*). It was certainly not compiled by mere guesswork. The fact that it was tried at all is impressive. The very existence of a table like Ulpian's represents a fine example of the practical way the Roman lawyers solved a problem that was theoretically almost insoluble with the means available at the time. Whether Ulpian himself was the originator of the table or not, is immaterial. As it is, it stands out as a monument to his generation because of the tacit presumption implicit in the construction of the table. It is the conviction that in the process of administering justice rational standards will have to be met.

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