Introduction

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1 Introduction

This book is a collection of studies on quantification over individuals and over degrees and the ways in which quantifiers in these domains interact. Such interactions are diverse, ranging from intervention effects to cross-domain parallels in composition and meaning. ‘Individual quantifiers’ are the traditional ‘generalized quantifiers’ that relate sets of individuals (Barwise & Cooper 1981). An individual quantifier like *every* holds of two sets of individuals when the first is a subset of the second (that the Canadians are a subset of the intrepid things in *Every Canadian is intrepid*). ‘Degree quantifiers’ relate sets of degrees. A degree quantifier like the comparative *-er* holds of two sets of degrees when the first is a subset of the second (that the degrees to which John is tall are a subset of the degrees to which Mary is tall in *Mary is taller than John*). The set of individuals and of degrees differ only in sort—they are both collections of semantically basic entities with no argument structure of their own. Further, both types of quantifiers are used to talk about quantity; both may be used to answer questions about how much or how many, suggesting that some notion of quantificationality characterizes meanings in both sortal domains. The parallels run so deep that for some quantifiers, parallel degree and individual quantificational analyses have been postulated, for example for the cardinal numerals *two*, *three*, etc., which may be taken to specify the cardinality of a set of individuals (Barwise & Cooper 1981) or to specify the maximal degree in the degree set corresponding to that cardinality (Krifka 1989, Landman 2004 and many others). Likewise, *many* and its comparative and superlative counterparts *more* and *most* have been analyzed as both individual quantifiers (Barwise & Cooper 1981, Westerståhl 1985b) and degree modifiers (Bresnan 1973, Hackl 2009). In the other direction, the comparative and superlative have been analyzed as relations between individuals rather than sets of degrees (Heim 1985, Bhatt & Takahashi 2011), bearing a stronger resemblance to individual quantifiers on this approach. Accordingly, individual and degree quantifiers are not blind to one another when found in the same syntactic environment: the Heim-Kennedy constraint (Kennedy 1999, Heim 2001) prohibits an individual quantifier from intervening between

a degree quantifier and the variable it binds. The contributions to this volume venture into the shifting frontier between individual and degree quantification and dissect its nuances, and generally reflect the steady encroachment of degree semantics into the territory of generalized quantifier theory that the past 20 years have witnessed. In the following two sections, I describe the theoretical background to this pursuit, first in relation to individual quantification in section 2, then in relation to degree quantification in section 3. Section 4 describes the contributions to this volume individually against this background.

2 Individual Quantifiers

Building on work by Lindenbaum & Tarski (1936) and Mautner (1946), Mostowski (1957) defines a ‘generalization’ of the standard universal ($\forall$) and existential ($\exists$) quantifiers of predicate logic. Such ‘generalized quantifiers’ are functions that map propositional functions to truth values subject to a ‘permutation invariance’ requirement, which I describe in more detail below. Propositional functions are predicates, such as *Canadian*, *intrepid*, etc., that map individuals to truth values. In these terms, the existential quantifier maps a propositional function F to ‘true’ if $F(x)$ is true for at least one value of x ; the universal quantifier maps F to ‘true’ if $F(x)$ is true for all values of x . But this system also accommodates quantificational notions like ‘there are three’ or ‘there are finitely many’, which maps F to ‘true’ if $F(x)$ holds for three or finitely many values of x respectively.

Mostowski discusses only quantifiers that denote properties of propositional functions, but Lindström (1966) observes that the features Mostowski attributes to generalized quantifiers extend fruitfully to relations between propositional functions. Mostowski and Lindström treat only formal logical calculi, but the significance of the notion of generalized quantifier for natural language was recognized by Montague (1973), Barwise & Cooper (1981), Keenan & Stavi (1986) and many others. Barwise and Cooper point out, in particular, that certain natural language determiners, such as *most*, cannot be defined at all as first-order quantifiers along the lines of $\forall$ and $\exists$. But as a generalized quantifier, *most* is characterizable as mapping a pair of propositional functions F and G to ‘true’ if G holds of more than half of the things F holds of. Barwise and Cooper define quantifiers as properties of sets rather than operators on propositional functions, taking first order predicates like *Canadian*, *intrepid*, etc., to denote sets (of Canadians, intrepid things, etc.). A few quantifier definitions along these lines are listed below.

- (1) For any sets A and B :
- a. $\llbracket \text{every}(A)(B) \rrbracket = \text{true}$ iff $A \subseteq B$
 - b. $\llbracket \text{some}(A)(B) \rrbracket = \text{true}$ iff $A \cap B \neq \emptyset$
 - c. $\llbracket \text{most}(A)(B) \rrbracket = \text{true}$ iff $|A \cap B| > |A - B|$
 - d. $\llbracket \text{four}(A)(B) \rrbracket = \text{true}$ iff $|A \cap B| = 4$

A sentence like (2a) translates into the logical form in (2b), where C is the set of Canadians and I the set of intrepid things. (2b) dictates that this sentence is true if and only if the set of Canadians is a subset of or equal to the set of the intrepid things ('or equal to' because the sentence does not require there to be any intrepid things other than Canadians).

- (2) a. Every Canadian is intrepid.
 b. $\llbracket \text{every}(C)(I) \rrbracket = \text{true}$ iff $C \subseteq I$

The mapping from the surface syntax to the logical form is not always trivial. In the simple cases above, the quantifier combines first with its syntactic sister, the nominal representing what is often called its 'restriction' (the set A in the definitions in (1)), then the result combines with the main predicate, often called the quantifier's 'nuclear scope' (the set B in (1)) (Heim 1983).

In other cases, though, the quantifier must be displaced from its surface position in order to derive a coherent semantic composition. In the sentence in (4a), for example, the verb *return* selects an individual-denoting object, yet the object position is filled by the quantifier *every book*, which denotes a function from sets to truth values. This combinatorial mismatch is rectified by displacement of the quantifier to a higher position, as illustrated in (4b). There, the sister to the quantifier contains a variable bound by an abstraction operator. The job of the abstraction operator is to derive a set of entities that make the underlying proposition true when substituted for the variable, in this case the set of things that Mary returned to the library.

- (4) a. Mary returned every book to the library.

The definition in (1a) specifies the truth conditions in (5) for the structure in (4b), namely that the set of books is a subset of the things Mary returned to the library.¹ Here, the sentence in (4a) is mapped to the format with which truth conditions are associated by displacement of the quantifier, a syntactic process referred to as ‘quantifier raising’ (May 1977, 1981, 1985, Beghelli & Stowell 1997, Heim & Kratzer 1998, Bruening 2001).

- (5) $\llbracket \text{every}(\mathbf{B}, \{x \mid \text{Mary returned } x \text{ to the library}\}) \rrbracket =$
 $\text{true iff } \mathbf{B} \subseteq \{x \mid \text{Mary returned } x \text{ to the library}\}$

Quantifiers display a number of logical properties relevant to the discussion of degree modifiers below. One is the permutation invariance property mentioned above. Following Mostowski, a permutation (more generally ‘transformation’) is a mapping of the entities that build the set-relata of quantifiers (the model’s ‘universe’) to another, not necessarily different set with the same cardinality. Linguistic applications are usually limited to automorphisms—a mapping from a set onto itself. This guarantees sameness of cardinality. If ϕ is such a mapping, then for any n -ary relation R (denoting a set of n -ary tuples), R_ϕ is defined as in (6a) as the ‘corresponding’ set in the image of the permutation. A quantifier Q is permutation invariant if $Q(A)$ has the same truth value $Q(A_\phi)$

1 Of course, the set $\mathbf{B}$ here is not naturally construed as the set of all books in the universe, but a set of relevant books, perhaps previously mentioned in the discourse context. See Westerstahl (1985a), von Stechow (1994), Cooper (1996), Stanley & Szabó (2000) and Stanley (2000, 2002), on context-dependent quantifier domain restrictions.

for any set A , extended in (6b) to n -ary quantifiers (Mostowski 1957, Lindström 1966, van Benthem 1983, 1984, 2002, Peters & Westerståhl 2006).

- (6) a. $\llbracket R_\phi \rrbracket = \{ \langle \phi(x_1), \dots, \phi(x_n) \rangle \mid \langle x_1, \dots, x_n \rangle \in R \}$
 b. $Q(A_1, \dots, A_n) \equiv Q(A_{1\phi}, \dots, A_{n\phi})$

By virtue of the definition in (6a), if, for example, the subset of the universe that I ('intrepid') denotes has four individuals in it, so does I_ϕ , but not necessarily the same individuals. Suppose *Mary* denotes the quantifier $\lambda P . P(\text{Mary})$ (per Montague 1973, the set of properties that Mary has). Then *Mary* is not permutation invariant, since the truth of *Mary*(I) ('Mary is intrepid') does not guarantee the truth of *Mary*(I_ϕ) (Mary might be in the set I but not the set I_ϕ). But the truth of *four*(C)(I) ('Four Canadians are intrepid', with *four* defined as in (1d)) guarantees the truth of *four*(C_ϕ)(I_ϕ) and vice versa: If C and I share four things, then again by virtue of (6a), the images of those four things under ϕ will be in both C_ϕ and I_ϕ , validating *four*(C_ϕ)(I_ϕ). If C and I do not overlap in four things, then neither will C_ϕ and I_ϕ . Intuitively, a quantifier that is permutation invariant does not care about the identities of things in the sets it relates; replacing those things with other things in a way that preserves the overall structure of the universe does not effect the interpretation of the quantifier.

Another property relevant to the comparison between individual and degree quantifiers in section 3 is monotonicity. Different quantifiers display different entailment patterns on their arguments (Ladusaw 1979, Barwise & Cooper 1981, van Benthem 1983, 1984, Peters & Westerståhl 2006). For example, the first argument of *every* may be replaced with a subset of that argument's denotation preserving truth. Since the French Canadians are a subset of the Canadians, the truth of (2a) guarantees the truth of (7a). But if we restrict the second argument similarly to a subset of the intrepid things, for example the things that are both intrepid and thrifty, there is no guarantee of preservation of truth; (2a) does not entail (7b).

- (7) a. Every French Canadian is intrepid.
 b. Every Canadian is intrepid and thrifty.

This means that *every* is downward (from superset to subset) entailing (i.e. 'downward monotone') in its first argument, but not in its second. Rather, it is upward entailing ('upward monotone') in its second argument, since (2a) entails (8) on the assumption that all intrepid things are mortal.

- (8) Every Canadian is mortal.

Not all quantifiers display the same entailment patterns; the quantifier *no*, for example, is downward entailing in its first argument ((9a) entails (9b)) but, unlike *every*, also downward entailing in its second argument ((9a) entails (9c) and not (9d)).

- (9) a. No Canadian is intrepid.
- b. No French Canadian is intrepid.
- c. No Canadian is intrepid and thrifty.
- d. No Canadian is mortal.

Other quantifiers are neither upward or downward entailing on one or more of their arguments. Such quantifiers are ‘non-monotone’. For example, *exactly three* entails neither downward nor upward on its first argument ((10a) entails neither (10b) nor (10c)), nor either downward or upward on its second argument ((10a) entails neither (10d) nor (10e)). *Exactly three* is non-monotone on both arguments.

- (10) a. Exactly three Canadians are intrepid.
- b. Exactly three French Canadians are intrepid.
- c. Exactly three North Americans are intrepid.
- d. Exactly three Canadians are intrepid and thrifty.
- e. Exactly three Canadians are mortal.

As Fauconnier (1975) and Ladusaw (1979) point out, downward entailing contexts license negative polarity items like *any* or *ever*. As predicted, these terms occur the argument positions of quantifiers shown to be downward entailing above, and not elsewhere.

- (11) a. Every Canadian I’ve ever met was (*ever) intrepid.
- b. No Canadian I’ve ever met was ever intrepid.
- c. Exactly three Canadians I’ve (*ever) met were (*ever) intrepid.

A final characteristic of quantifiers relevant to the comparison with degree modifiers below is the ‘conservativity’ property. Barwise & Cooper (1981), Higginbotham & May (1981), van Benthem (1983, 1984), Westerstål (1985b), Keenan & Stavi (1986), Peters & Westerstål (2006) and others describe a restriction on the interpretation of natural language determiners that excludes many logically possible determiner denotations. The conservativity restriction dictates that a natural language quantifier *Q* holds of sets *A* and *B* if and only if it also holds of *A* and $A \cap B$.

- (12) For any sets A and B and any determiner Q :
- $$Q(A)(B) \equiv Q(A)(A \cap B)$$

Every is conservative because *Every Canadian is intrepid* is true if and only if *Every Canadian is an intrepid Canadian* is true (to be an intrepid Canadian is to be in the intersection of the sets that *intrepid* and *Canadian* denote). Westerstål (1985b) mentions that the ‘Rescher’ quantifier defined as R in (13a) is not conservative, and consequently cannot function as the denotation of any natural language determiner. If, for example *more* denoted the Rescher quantifier, (13b) would mean what (13c) means. But it cannot; (13b) is ungrammatical without a (potentially implicit) *than*-clause, in which case it expresses something very different from (13c).

- (13) a. $\llbracket R(A)(B) \rrbracket = \text{true iff } |A| > |B|$
 b. *More Canadians are linguists.
 c. There are more Canadians than linguists.

With this broad overview of natural language quantifiers and some of their more salient syntactic and logical properties in mind, I turn below to a discussion of degree quantifiers and parallels to individual quantifiers.

3 Degree Quantifiers

Seuren (1973), Cresswell (1976), Klein (1980), and others point out that natural language predicates that encode a gradable quality can be characterized as descriptions of an extent, or degree, of that quality. On one view, this degree functions as an argument of the gradable predicate, so that e.g. *tall* is a relation between an individual (x in (14a)) and a degree of height (d in (14a)) (Seuren 1973, Cresswell 1976). The definition in (14a) models this denotation as a set of individual-degree pairs meeting the condition that x has height d . On another view, the gradable predicate is not a relation but a function that maps an individual argument to a degree, so that *tall*, as illustrated in (14b), combines with an individual x and returns x ’s height, a degree (Bartsch & Vennemann 1973, Kennedy 1999). I speak here primarily in terms of the relational view spelled out in (14a) below.

- (14) a. $\llbracket \text{tall} \rrbracket = \{ \langle x, d \rangle \mid \text{tall}(x, d) \}$
 b. $\llbracket \text{tall} \rrbracket = \lambda x . x\text{'s height}$

A primary difference between degrees and other entities, that has certain ramifications for the analysis of degree quantifiers, is that degrees are inherently ordered. Every degree carries information about how it is related to other degrees. Concretely, Cresswell (1976) defines ‘degree’ as a pair consisting of a coordinate u and an ordering relation $>$.² $\mathcal{F}(>)$ denotes the ‘field’ of $>$, the set of things that either stand in the $>$ relation to something, or that something stands in the $>$ relation to.

- (15) A DEGREE (of comparison) is a pair $\langle u, > \rangle$, where $>$ is a relation and $u \in \mathcal{F}(>)$.
(Cresswell 1976, p. 266)

A set of such pairs is a ‘scale’. The particular relation $>$ at hand is determined by the gradable predicate itself. *Tall* determines a scale of height, *intrepid* of intrepidity, etc. Note that Cresswell’s definition leaves some leeway for interpreting u . While u is commonly treated as an abstract coordinate, Cresswell himself mentions the possibility that u stands for an individual, which is then ranked with other individuals directly by the gradable relation, rather than through a relation to a degree. Contributions to the present volume by Alan Bale and Roger Schwarzschild pursue versions of this approach. Whether u represents a coordinate or an individual, the pairs that (15) defines are semantically basic elements with no argument structure of their own, like individuals, meaning that this definition of ‘degree’ does not impact the parallels in semantic type described below between individual quantifiers and degree quantifiers. In fact, it may go some way toward explaining the sortal difference between individuals and degrees. Individuals are singleton entities, degrees tuples. It seems reasonable that languages use a different vocabulary for quantification over the two sorts. I continue to use the variable symbol d to stand for the pairs illustrated in (15).

When the individual argument of a degree relation is saturated, we are left with a set of degrees. That is, *Mary is tall* is interpreted as the set of degrees to which Mary is tall. This set is typically, but not universally, treated in degree semantic studies as downward monotone, meaning that if Mary is tall to some degree, she is tall to every lesser degree as well. To put it another way, the formula ‘ $\text{tall}(x, d)$ ’ is read ‘ x is at least d -tall’. Not all analyses of degree quantifiers proceed from the premise that degree scales are downward monotone

2 For more on the ontology of degrees, see Landman & Morzycki (2003), Anderson & Morzycki (2015) and Snyder (2017), who analyse degrees as ‘kinds’ in the sense of Carlson (1977a) and Chierchia (1998).

(Schwarzschild & Wilkinson 2002, Zhang & Ling 2015). Those that do typically exploit the fact that if Mary is taller than John, the set of degrees to which Mary is tall properly contains the set of degrees to which John is tall. This fact plays an important role in the analysis of the comparative and superlative, to which I now turn.

Heim (1985) describes an approach to the comparative that makes it a relation between two individuals and a gradable predicate. This is referred to as the ‘phrasal comparative’ because it relates nominal phrases (DPs), as opposed to the ‘clausal comparative’ discussed below. I define the phrasal comparative as in (16), departing from Heim’s definition somewhat to highlight parallels to individual quantifiers discussed in section 2 (see footnote 4 to follow). It holds when the set of degrees to which the first individual bears the scalar relation is a proper subset of the set of degrees to which the other individual bears the relation.

$$(16) \llbracket -er(x_e)(P_{\langle e, \langle d, t \rangle \rangle})(y_e) \rrbracket = \text{true iff } \{d \mid P(x, d)\} \subset \{d \mid P(y, d)\}$$

This definition captures the meaning of a comparative sentence like *Mary is taller than John* directly as a relation between the two individual arguments *Mary* and *John* (on this view, the ‘standard marker’ *than* is vacuous). Bhatt & Takahashi (2011) claim that only this use is attested in Hindi-Urdu and Japanese, whereas English displays the ‘clausal comparative’ (following Lechner 2001, 2004). The argument structure illustrated in (16) is unexpected for a quantificational expression. At first glance, *-er* looks more like a transitive verb than a quantifier; the individual quantifiers discussed in section 2 never relate actual individuals, only sets.³ However, although the phrasal comparative relates individuals, it qualifies as permutation invariant by the following line of reasoning. On the basis of the definitions in (6), when we ask whether *-er* in (16) is permutation invariant, we are asking whether an equivalence holds between $-er(x)(R)(y)$ and $-er(\phi(x))(R_\phi)(\phi(y))$. Suppose x bears R to the degrees 1 and 2, and y bears R to 1, 2 and 3 (for the sake of explication; whether it is reasonable to equate degrees with numbers is another matter I discuss below). By the definition of R_ϕ , $\phi(x)$ bears the R_ϕ relation to $\phi(1)$ and $\phi(2)$, while $\phi(y)$ bears it to $\phi(1)$, $\phi(2)$ and $\phi(3)$. Since the set of things that $\phi(x)$ bear R_ϕ to (namely $\{\phi(1), \phi(2)\}$) is a subset of the set of things that $\phi(y)$ bears R_ϕ to (namely $\{\phi(1), \phi(2), \phi(3)\}$),

3 Bhatt & Takahashi (2011) make this point in footnote 19, p. 606, where they attribute it to the syntactic type of individual quantifiers as determiners.

$\phi(3)\}$), the truth of *-er* is preserved across the permutation, and the phrasal comparative qualifies as quantificational in this sense.⁴

Another option for the analysis of the comparative characterizes it as a relation between degree set descriptions that are constructed in the syntax. This view seems to be required for sentences like (17), where the *than*-clause contains a sentence fragment. On the 'clausal' analysis of the comparative, (17) is syntactically analysed as containing an elided predicate in the *than*-clause identical with the main clause predicate (Bresnan 1973, Seuren 1973, Cresswell 1976, Lechner 2001, 2004).

(17) Mary is taller than John is.

In Seuren (1973), Cresswell (1976) and elsewhere, both the main clause and the *than*-clause are construed as sets of degrees related by the comparative morpheme *-er*. The comparative morpheme *-er* functions as the quantifier (qua relation between sets) and the *than*-clause as its restriction (Bresnan 1973, 1975, Carlson 1977b). Much of Heim's paper cited above concerns the question of whether the phrasal analysis can be reduced to the clausal analysis. Hankamer (1973), Hoeksema (1983) and others detail syntactic differences between the phrasal and clausal comparative, while Bhatt & Takahashi (2011) show that some languages dispose only of a syntactically phrasal comparative construction. The extent to which the syntactic distinction aligns with a semantic one remains under debate. Contributions to the present volume by Barbara Tomaszewicz-Özakın and by Bogal-Allbritten and Coppock discuss this issue, as I describe in more detail in section 4. In the clausal comparative, the mapping from the surface structure, in which *-er* is a suffix of the main clause degree predicate, to the logical form is, as in the case of non-subject individual quantifiers illustrated in (4b), not trivial. Quantifier raising analogous to what we see with individual quantifiers in (4b) displaces the degree quantifier *-er* and its restriction (the *than*-clause), to a position adjoined to the main clause (Seuren

4 Heim and others define the comparative in terms of the 'greater than' relation between maximal degrees in the relevant degree sets, along the lines of (i):

(i) $\llbracket \text{-er}(x_e)(P_{(e,(d,t))})(y_e) \rrbracket = \text{true iff } \max\{d \mid P(x, d)\} > \max\{d \mid P(y, d)\}$

For this definition to qualify as permutation invariant, we must ensure that the images of the degrees under ϕ are ranked with respect to $>$ in the same order as the respective degrees. That is, we must carry some of the structure of the original model into the permutation model. Van Benthem (1982) discusses this issue in connection with both gradable adjectives and temporal modifiers, both of which make reference to linear orders (of degrees and times) which are preserved under permutation. See van Benthem (1983, 1986) and Peters & Westerståhl (2006) on restrictions on the transformations relevant to the notion of logicity.

1973, Postal 1974, Cresswell 1976, Williams 1977, von Stechow 1984, Heim 1985, 2001).⁵ This derives the structure in (18b), where as in the case of the individual quantifier in (4b), displacement of the quantifier (with its restriction) goes hand in hand with abstraction of a degree predicate over the main clause (arguably *than* plays this role in the restriction). Quantifier raising creates an antecedent for ellipsis in the *than*-clause.

(18) a. Mary is taller than John is.

The two S' constituents denote the set of degrees to which John is tall (call it A) and the set of degrees to which Mary is tall (call it B). The comparative morpheme *-er* takes these sets as arguments and is true when B contains degrees not included in A , or, put another way, A is a proper subset of B , schematized in (19) Heim (2006a).

5 The works cited here have in common that they attribute scopal flexibility to *-er*, though the mechanism takes a variety of shapes. Postal and Seuren attribute the ambiguity to different base structures, not different logical derivatives. von Stechow (1984) points out that the earlier works incorrectly leave the *than*-clause in situ. Bhatt & Pancheva (2004) claim that the degree quantifier *-er* moves but the *than*-clause is merged with *-er* in its derived position. Bresnan (1975) and Chomsky (1977) present syntactic evidence for A' movement of a covert operator in the *than*-clause, which they associate with degree abstraction there.

$$(19) \llbracket -er(A)(B) \rrbracket = \text{true iff } A \subset B$$

As spelled out in (20), the truth conditions for (18a) bear a certain resemblance to the truth conditions for the universal individual quantifier example in (4a) *Mary returned every book to the library* (recall **B** designates the set of books in that example).

- (20) a. $\{d \mid \text{tall}(\text{John}, d)\} \subset \{d \mid \text{tall}(\text{Mary}, d)\}$
 b. $B \subseteq \{x \mid \text{return}(\text{Mary}, x, \text{the-library})\}$

Both the phrasal comparative as defined in (16) and the clausal comparative as defined in (19) put two degree sets in the proper subset relation (the clausal comparative directly and the phrasal comparative indirectly). In spite of the similarity, the comparative *-er* and the individual quantifier *every* differ in both the kind of subset relation they invoke (proper in (20a) and non-proper in (20b)), and in the things they quantify over. The morpheme *-er* relates sets of degrees, while *every* relates sets of individuals. Again, degrees and individuals are both semantically atomic entities. They differ in 'sort'. Both quantifiers potentially involve quantifier raising in the derivation of a coherent logical form.

The degree quantifier *-er* as defined in (19) straightforwardly displays the permutation invariance property typical of natural language quantifiers, for reasons described above. Suppose for some sets of degrees A and B that A is a subset of B , so $-er(A)(B)$ holds. And suppose some permutation ϕ maps the all the degrees in the universe to other things in a universe with the same cardinality. Since B_ϕ consists by definition of the images under ϕ of all the things in B , and B contains all the things in A , inevitably B_ϕ contains the images under ϕ of all the things in A , which is just A_ϕ . By the same reasoning, if $A \not\subset B$ then $A_\phi \not\subset B_\phi$. So $-er(A)(B) \equiv -er(A_\phi)(B_\phi)$.

As expected in light of the similarity in meaning between *-er* and *every*, the degree-set relata of *-er* display the same entailment pattern as the individual-set relata of *every* (Seuren 1973, Hoeksema 1983). Suppose John is taller than Jane. By virtue of the downward monotonicity of degree predicates discussed above, the set of degrees to which Jane is tall is a subset of the set of degrees to which John is tall. On this assumption, (21a) entails (21b), where we replace the set of degrees to which John is tall with a subset—the set of degrees to which Jane is tall. The degree quantifier *-er*, then, is downward entailing in its restrictor argument (the *than*-clause). Since John is taller than Jane, Mary being taller than John guarantees she is taller than Jane.

- (21) a. Mary is taller than John is.
 b. Mary is taller than Jane is.

Also like *every*, *-er* is upward entailing in its nuclear scope argument. Replacing *Mary* in (22a) by someone taller than her (Julie in (22b)) preserves truth.

- (22) a. Mary is taller than John is.
 b. Julie is taller than John is.

Accordingly, *-er* licenses negative polarity items in its restrictor argument but not its nuclear scope argument.

- (23) Mary will (**ever*) be taller than John will ever be.

Recent developments that bear on these monotonicity patterns deal with a perennially troublesome interaction between degree and quantification involving the interpretation of individual quantifiers in *than*-clauses. The monotonicity of degree predication (the fact that if $R(x, d)$ holds, then it holds for every $d' < d$ as well) predicts an unattested interpretation for sentences like *Mary is taller than every boy is*. If *than every boy is* [*tall*] denotes the set of degrees to which every boy is tall, then this set should include only the degrees to which the shortest boy is tall, since these are the only degrees that are common to the heights of all the boys. Consequently, the sentence should mean that Mary is taller than the shortest boy, but in fact it requires her to be taller than the tallest (von Stechow 1984). In principle, giving ultra wide scope to *every boy* solves the problem (then it means that for each boy, Mary is taller than him), but independent syntactic factors militate against this approach. Among other issues, *than*-clauses are typically barriers for extraction, as (24) demonstrates, from Larson (1988).

- (24) *Who_{*i*} is Felix taller than Moe persuaded *t_i* that Max is?

One type of solution to this problem involves various devices that give the quantifier internal to the *than*-clause wide scope without actually removing it from the *than*-clause (Larson 1988, Fleisher 2016, Nouwen & Dotlačil 2017). Nicholas Fleisher's contribution to the present volume, which analyses the effect as a kind of pair list reading (and *than*-clauses as a kind of question) is an example of this approach. This approach does not involve drastic revisions to the standard degree-semantic view of what the degree quantifier *-er* means nor the monotonicity of degree scales. Another type of solution to this conundrum

is ‘interval semantic’. Schwarzschild & Wilkinson (2002) present an analysis of the comparative along the following lines, developed in Heim (2006b), Gajewski (2008), van Rooij (2008), Schwarzschild (2008), Beck (2010), Abrusán & Spector (2010), Dotlačil & Nouwen (2016) and elsewhere.⁶ Rather than denoting the set of degrees that all the boys have in common, the *than*-clause in *Mary is taller than every boy is* denotes a set containing the maximal heights of the individual boys in question. In turn, the main clause denotes a set containing the height of Mary. These are degree ‘intervals’ (or ‘segments’, as Schwarzschild calls them in this volume). The comparative says that the main clause interval is separated from the *than* clause interval by a positive difference, so the first contains only degrees higher than all the degrees in the second. On this view, the *than*-clause is no longer predicted to entail downward, and indeed, Schwarzschild and Wilkinson argue that the conclusion that *than*-clauses are downward monotone is premature, on the basis of facts like those in (25). The entailment from *exactly 7* to *at least four* that in (25a) illustrates, for example, is not reversed in the *than*-clauses in (25b).

- (25) a. Exactly 7 of my relatives are rich $\rightarrow$ At least 4 of my relatives are rich.
 b. John is richer than at least 4 of my relatives were $\nrightarrow$
 John is richer than exactly 7 of my relatives were.

In this volume, Linmin Zhang spells out an interval theoretic solution to the particular problem of non-monotone quantifiers (such as *exactly two*) in *than*-clauses, while Roger Schwarzschild presents an interval theoretic approach to the analysis of differentials in intensional contexts.

When we turn to the conservativity property, *-er* does not behave entirely parallel to individual quantifiers. The meanings of the two quantifiers *-er* and *every*, though similar, differ in the kind of subset relation they denote, proper in (20a) and non-proper in (20b). If Mary and John are exactly the same height, we cannot assert that Mary is taller than John. That sentence requires that the set of degrees to which Mary is tall contains some degrees not in the set of degrees to which John is tall, i.e., that the latter is a proper subset of the former. But if Mary and John are the same height, we can say (26a), an ‘equative’ degree construction discussed at length by Jessica Rett in the present volume. Example (26a) is true when Mary is tall to all the degrees to which John is tall. Although it implies that she is no taller than him, this implicature can be defeated, as in (26b). Consequently, it appears that we can give equative *as* the denotation in

6 Alrenga & Kennedy (2014) import certain elements of the interval semantic approach into a traditional monotone degree semantics.

(26c), which attributes the truth conditions in (26d) to (26a). These truth conditions are exactly parallel to *every*, albeit in the degree domain.⁷

- (26) a. Mary is as tall as John.
 b. Mary is as tall as John is, in fact she's even taller.
 c. $\llbracket \text{as}(A)(B) \rrbracket = \text{true iff } A \subseteq B$
 d. $\{d \mid \text{tall}(\text{John}, d)\} \subseteq \{d \mid \text{tall}(\text{Mary}, d)\}$

While equative constructions may represent a better parallel to the meaning of *every* than comparatives, there does not seem to be a counterpart to *every* that, in parallel to comparative *-er*, denotes proper subsethood. Such a counterpart in place of *every* in (20b) would entail that Mary returned something to the library other than and in addition to the books in question. An explanation for this missing counterpart to *every* presents itself in the form of the conservativity constraint. As discussed in section 2, conservativity prohibits a quantifier from saying anything about the members of *B* that are not also members of *A* in the schemas in (1). But this is precisely what the proper subset relation does: it holds precisely when there are members of *B* that are not also members of *A*. The proper subset relation is an impossible determiner meaning because it violates conservativity.

But then the fact that *-er* denotes the proper subset relation means that degree quantifiers (insofar as *-er* is representative) are not subject to conservativity. On one hand, this might be taken to represent a difference between individual and degree quantifiers. On the other hand, it has been pointed out that conservativity is a restriction that holds only of determiners, not of quantifiers in general. A clear case of a non-conservative quantifier is *only*. The sentence *Only Canadians are intrepid* is not equivalent to *Only Canadians are intrepid Canadians*. Von Stechow (1997) reasons that *only* is not a determiner, but an adverb, and therefore exempt from the conservativity requirement. This explanation of course carries over to *-er* and other degree quantifiers, as these are not determiners. But on the other hand, it supports the idea that conservativity is not foremost a restriction on quantifier meanings, but rather a restriction on the meanings of determiners (as a syntactic category) with no fundamental connection to quantification, other than the fact that determin-

⁷ Equative constructions are usually understood by default to mean *exactly as much as*. While a pragmatic account of this strengthening suggests itself (if the speaker knew Mary was taller than John she would say so), a reviewer of this chapter points out that *taller than* emerges as an alternative to *as tall as* when focus falls on *as*, as when we say *Mary is not as tall as John, she's TALLER than him*. This is probably made possible by fact that the equative and comparative are duals, as Rett (this volume) points out.

ers often have quantificational meanings, as do adverbs and ‘degree modifiers’ (to put it in syntactic terms). If that is so, then the fact that degree quantifiers are not subject to conservativity does not represent a difference between them and individual quantifiers.

Another approach to the conservativity puzzle that *-er* presents is spelled out in Bhatt & Pancheva (2004). Following the suggestion of Fox (2002) and Sportiche (2005), they claim that conservativity is an epiphenomenon of the copy theory of movement. When a quantifier moves, it leaves a copy of its restriction in its θ -position that restricts the value for the variable in that position. Consequently, *Every Canadian is intrepid* is interpreted as *Every Canadian is Canadian and intrepid*, mimicking the conservativity equivalence in (12) simply as a result of quantifier raising. What is special about degree quantifiers such as *-er*, they claim, is that their restriction (the *than*-clause), can be inserted into the derivation after movement of the quantifier, so that it is not present in the base position. This makes non-conservative semantic relations available as denotations for degree quantifiers.

We have seen that degree relations are typically assumed to be downward monotone, in the sense that $R(x, d)$ implies $R(x, d')$ for all d' lower than d , and that this plays a role in the interpretation of degree quantifiers but not individual quantifiers, which do not have ordered domains. Another aspect of degree scales responsible for differences between degree and individual quantifiers is the fact that degree scales may be ‘dense’, meaning that for any two degrees on the scale, there is a third between them. By some accounts, all degree constructions involve dense scales (Fox & Hackl 2006), even those that measure out the cardinality of collections individuals, which I discuss in more detail below. The density of degree scales means that as soon as two scales overlap in two degrees, they overlap in infinitely many. For this reason, if cardinal quantifiers like *four* count the elements in the intersection of the two sets they relate (or compare their cardinalities like *most*), then they make little sense when applied to degree sets. Two scales that intersect in four points also intersect in the infinitely many points between them, so *exactly four* as a description of the cardinality of the intersection of two sets of degrees will never be true, and *at least four* is indistinguishable from the infinitely many other cardinal quantifiers above ‘one’. Likewise, the claim that most of the degrees in set *A* are also in set *B* is not useful if both *A* and *B* contain infinitely many degrees. Such degree quantifiers are not logically impossible, but their communicative usefulness is so limited that it comes as no surprise that they are unattested.

These remarks suggest that hypothetical quantifiers that specify the cardinality of degree sets (i.e., that ‘count’ degrees) are not useful because of the density of degree scales. But this does not mean that degree semantics is entirely

uninvolved in counting. A variety of evidence in fact indicates that cardinality is merely one kind of degree scale, and numbers, rather than being degree quantifiers like the comparative, are names for degrees.

Consider the fact that the comparative has ‘quality’ and ‘quantity’ variants (Gawron 1995) that correspond to comparisons of what I have called degree and cardinality above.

- (27) a. Mary read a more interesting book than John. [quality]
 b. Mary read more books than John. [quantity]

In parallel to (20a), (27a) denotes the proper subset relation between the set of degrees to which the book that John read is interesting and the set of degrees to which the book that Mary read is interesting. If *more* is a quantifier in (27b) corresponding to *-er* in (27a), then the two relata in (27b) are sets of degrees that correspond in some way to the number of books John and Mary read. That is, it is possible to make comparisons of cardinality using the same morphosyntactic machinery that languages use to make comparisons of degree. The same parallel can be seen in the superlative (28) and other degree modifiers like *very*, which implies a high degree of cardinality (29), as well as in the equative construction (not shown).

- (28) a. Mary read the most interesting book. [quality]
 b. Mary read the most books. [quantity]

- (29) a. Mary read a very interesting book. [quality]
 b. Mary read very many books. [quantity]

The quantity use of these degree quantifiers involves a plural (or mass) noun, whose cardinality (or ‘amount’) is being assessed. Plural nouns describe plural individuals—sums of atoms (Link 1983, Lønning 1987). Standard accounts of quantity constructions (the b-examples above) claim that, like gradable predicates, plural nouns also have a degree argument that corresponds to the cardinality of the plural individual—the number of atoms it has. Cresti (1995), Heycock (1995), Hackl (2000, 2009), Landman (2004), Wellwood (2015) and others claim that the degree argument is introduced by the modifier *many* (before count nouns) or *much* (elsewhere), which is subsequently deleted in some contexts. Others postulate a covert measure function for this purpose (Landman 2004, Kayne 2005, Rett 2006, Schwarzschild 2006, Scontras 2014, Snyder 2017). Cresswell (1976), Krifka (1989), Corver (1997), Solt (2015) and others claim that the degree argument is introduced directly by the plural noun,

and that *many/much* merely morphosyntactically hosts the degree quantifier morphology when necessary.

On any account, a phrase like (*many*) *books* denotes a relation between a plural individual and a degree, which holds when the plural individual has the property *books* and the degree is its cardinality, as illustrated in (30). Compare the definition of (*many*) *books* in (30) with that in (14a) for *tall*.

$$(30) \llbracket (\text{many}) \text{ books} \rrbracket = \{ \langle x, d \rangle \mid \text{books}(x) \ \& \ |x| = d \}$$

Just as the quality comparative in (27a) says that the set of degrees corresponding to how interesting the book was that John read is a subset of the set of degrees corresponding to how interesting the book was that Mary read, spelled out in (31a), the quantity comparative in (27b) says that the set of degrees corresponding to the number of books that John read is a subset of the set of degrees corresponding to the number of books Mary read, spelled out in (31b). It appears that the language itself does not draw any sharp boundary between ‘quality’ and ‘quantity’ comparisons. Both involve abstraction over a degree argument.

$$(31) \text{ a. } \{d \mid \exists x [\text{John read } x \ \& \ \text{book}(x) \ \& \ \text{interesting}(x, d)]\} \subset \{d \mid \exists x [\text{Mary read } x \ \& \ \text{book}(x) \ \& \ \text{interesting}(x, d)]\} \\ \text{ b. } \{d \mid \exists x [\text{John read } x \ \& \ \text{books}(x) \ \& \ |x| \geq d]\} \subset \{d \mid \exists x [\text{Mary read } x \ \& \ \text{books}(x) \ \& \ |x| \geq d]\}$$

The quantity constructions above compare the cardinality of sets of entities using the morphosyntactic components of degree comparison seen in the quality counterparts, again suggesting that cardinality and degree are not unrelated syntactically or semantically. It is particularly revealing that the term *most* occurs in both quality and quantity degree constructions, since *most* has always been held to be a paradigmatic case of an individual quantifier with set relata, since its meaning cannot be formulated in first order logic. The observations above might be taken to show that there is no determiner *most*. Rather, *most* is none other than the superlative morpheme *-est* in combination with a morphological host *much* (contracted to *mo-*) which may or may not play a semantic role in the composition of *most*, as mentioned above.

This is just what Hackl (2009) claims. He attributes to *-est* the meaning in (32), following Heim (1999) and others. Superlative *-est* combines with a set of entities *C*, a degree relation *R*, and an individual *x*. *C* represents a ‘comparison’ class of individuals we are comparing *x* to (we presuppose that *C* is not empty). *R* is a relation like *tall* or *interesting* that relates an individual to a degree. The

morpheme *-est* holds of these relata when for each member y of C (other than x itself), x bears R to a greater degree than y does. The function *max* applies to a set of degrees and returns the greatest degree in that set (the degree that entails all the others on the relevant scale).

$$(32) \llbracket -\text{est}(C_{\langle e,t \rangle})(R_{\langle d, \langle e,t \rangle \rangle})(x_e) \rrbracket = \text{true iff} \\ \forall y \in C[y \neq x \rightarrow \max\{d \mid R(d)(x)\} > \max\{d \mid R(d)(y)\}]$$

Consider now the quality superlative example in (28a). If we parse *-est* as adjoined to the NP *interesting book* as illustrated in (33), then that NP identifies R in (32). Mary identifies x and C is interpreted as a set of some salient books. As in other examples of degree constructions above, an operator derives a degree predicate over a constituent containing a degree argument, here the NP *interesting book*. The superlative morpheme *-est* applies to this NP to derive a set of individuals x that have the degree property *interesting book* to a greater degree than any alternative y in C does. The definite article *the* picks out the unique item in this set, which is a singleton set by virtue of the meaning of *-est*, which in turn functions as object of the verb *read*. This is referred to as the ‘absolute’ reading of the superlative; it picks out the absolute most interesting book (from the alternatives in C).

As is well known, another reading is available for (28a), often referred to as a ‘relative’ reading. Here, we compare not books but rather Mary to others in terms of how interesting the books were that they read. Szabolcsi (1986), Heim (1999) and others claim that this reading is derived by movement of the superlative morpheme *-est* to a higher position, just as *-er* moves in the comparative (but see Farkas & Kiss 2000, Sharvit & Stateva 2002 and Coppock & Beaver 2015 for analyses that do not involve movement). In this case, we abstract a degree relation over the entire VP, rather than just the NP, and we understand *C* as a set of people who read interesting books. This analysis is illustrated in (34). Note that it requires us to interpret the definite article as indefinite or altogether vacuous, since the different values of *y* in the end formula below read different books. I assume an existential quantifier over *z* is introduced at the level of VP (Heim 1983, Diesing 1992, Chung & Ladusaw 2004).

The interpretation of the quantity superlative in (28b) is just that found in the ‘relative’ quality superlative reading of (28a) diagrammed in (34). Here, the gradable property *R* is abstracted over the degree argument that the plural noun *books* introduces (on some accounts via the modifier *many*, as mentioned above), and the other argument of *books* is a plural individual—an algebraic

sum of books. That is, *most* is functioning as a superlative here, not as an individual quantifier, as illustrated in (35).

Now, Hackl (2009) points out that the reading of *most* corresponding to the generalized quantifier meaning in (1c) can be characterized as the ‘absolute’ superlative counterpart to the relative construction in (28b), which his analysis identifies with a certain scope for *-est* (see also Szabolcsi 2012). A sentence like (36a) has the structure in (36b) on this view.⁸ As in the absolute quality superlative in (33), *est* has scope over just the NP object in (36b). It in turn derives an NP that compares a quantity of books *z* with all other non-equal quantities in the contrast set *C*. Hackl points out that if we take ‘non-equal’ to mean ‘non-overlapping’, the plurality *z* must have a cardinality that is more than half of

8 The phrase *most books* has a generic flavor that makes it somewhat awkward in object position in existential contexts. The addition of *of the* resolves this clash. See Schwarzschild (2006), Scontras (2014), Solt (2015), and Wellwood (2015), among others, on a more articulated internal structure for partitive constructions that addresses these issues.

the total number of books in C . The reason is that if it didn't, there would be an alternative plurality of books y comprising the rest of the books, that has a greater cardinality than z . The plurality z then must comprise more than half of the relevant books, as *most* does on the meaning the generalized quantifier analysis in (1c) attributes to it.

(36) a. Mary read most books.

Hackl's analysis removes *most* from the pantheon of generalized quantifiers as relations between sets of individuals. Solt (2011) and Szabolcsi (2012) develop this analysis of *most*. Dobrovie-Sorin (2013, 2015) and Dobrovie-Sorin & Giurgea (to appear), on the other hand, claim that superlative *most* only has a relative reading, but that a bona fide generalized quantifier *most* exists alongside superlative *most*. The generalized quantifier *most*, they claim, only occurs with count nouns, explaining the pattern in (37). It is unclear why Hackl's absolute reading for superlative *most* would be incompatible with a mass noun as in (37b) (or, to the extent it is compatible, why this context requires the support of *of the*; see footnote 8).

- (37) a. I am sure most men will arrive late.
 b. *I am sure most wine will be delivered late.

Whether or not a generalized quantifier *most* exists alongside the superlative, the superlative *-est* defined in (32), that underlies *most* on Hackl's analysis, is quantificational in the logical sense, since it is permutation invariant insofar as the 'greater than' relation among degrees is preserved in the permutation, as discussed in footnote 4. A permutation of the universe that respects the condition that $\phi(d) > \phi(d')$ whenever $d > d'$ ensures that the superlative relation between an individual and a property is preserved under the permutation.⁹ Like the phrasal comparative, then, *-est* has the primary logical property of quantifiers, even though it has an individual argument.

Further, the notion of cardinality-as-degree represents another potential vehicle for the analysis of numeral modifiers like *four*, analyzed as intersective quantifiers in generalized quantifier theory. If *books* has (perhaps in combination with a covert *many*) a degree argument that tracks its cardinality (the number of atoms in the plurality), then a numeral modifier like *four* might be analyzed as a name for that degree. That is, in the expression *four books*, *four* saturates the degree argument slot of *books*. On this view, rather than denoting relations between sets, numbers denote degrees.

The issue of whether natural languages have degree-referring expressions is a contentious one. One strand of this debate goes back to Frege's (1884) claim, popularized by Dummett (1973, 1991), that the word *four* in (38) names a number. Frege maintains that this fact represents evidence for the existence of numbers external to language. If (38) is true, then *four* has a referent, which is equated there with the referent of *the number of Jupiter's moons*.¹⁰

- (38) The number of Jupiter's moons is four.

A number of philosophers find fault with this argument's background assumptions about what (38) means (Higgins 1979, Moltmann 2013, Snyder 2017). Further, Bach (1986) expresses reservations about the validity of drawing metaphysical conclusions from linguistic evidence, but reaffirms the need for what Montague (1969) calls 'certain metaphysical entities' in a framework for natu-

9 As mentioned in footnote 4, this equivalence is easier to see if we define the superlative using the subset relation rather than using the $>$ relation with maximalization, as in (32).
 (i) $\llbracket \text{-est}(C_{(e,t)})(R_{(d,(e,t))})(x_e) \rrbracket = \text{true iff } \forall y \in C[y \neq x \rightarrow \{d \mid R(y, d)\} \subset \{d \mid R(x, d)\}]$

10 Jupiter turns out to have dozens more moons than the four largest observed originally by Galileo.

ral language interpretation. That is, *four* may name something in the model of the world that linguistic structures are interpreted with respect to, but whether the things that populate that model have counterparts in reality is not resolved by the linguistic facts. See Wellwood (this volume) on experimental evidence concerning the question of the primacy of metaphysics or conceptual models in linguistic cognition. I expand on the the notion of numbers as degree names below.

Other analyses treat bare numerals as predicate modifiers or as predicates that are composed by a rule of predicate modification with a noun to restrict its range.¹¹ That is, they restrict the argument of the noun to pluralities with the specified cardinality (Partee 1987, Kamp & Reyle 1993, Kadmon 1993, Krifka 1999, Winter 2001, Ionin & Matushansky 2006, Geurts & Nouwen 2007). This approach raises the question of what the proper analysis of specificational sentences like (38) should be, to which I also return below.

Another possible candidate for a degree name is found in ‘measure phrases’ such as *six feet* in examples like (39a) (Cresswell 1976, von Stechow 1984, Gawron 1995, Heim 2001, Matushansky 2002). If *tall* has a degree argument, a natural semantic role for *six feet* is that of name for that degree argument, so that *tall* as defined in (14a) composes with the two arguments in (39a) to yield the predicate logical formula in (39b).

- (39) a. Mary is six feet tall.
b. $\text{tall}(\text{Mary}, \text{six feet})$

Other analyses cast measure phrases as predicate modifiers (Klein 1980, Hackl 2000, Landman 2004, Schwarzschild 2005, Kennedy 2013, 2015, Scontras 2014, Snyder 2017). The analyses of Klein and Kennedy explicitly make them degree quantifiers, a point I return to below. The body of research cited above—collectively and to a greater or lesser extent individually—connects the form and meaning of measure phrases like *six feet* or *six pounds (of rice)* with that of bare numerals like *six* in *six cookies*, by virtue of the premise that the latter contains a covert measure function measuring out cardinality (again covert *many* in some analyses). All these accounts seek to decompose measure phrases into their component parts, which compose semantically in lockstep with their syn-

11 Ionin & Matushansky (2006) point out that these two modes of combination (function application vs. predicate modification) are not equivalent when it comes to the composition of complex numerals like 200, since *200 years* does not describe a plurality of years that has both cardinality two and cardinality 100. They analyze numerals explicitly as predicate modifiers, i.e., functions of type $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$.

tactic composition (though these accounts do not necessary agree on what the syntactic composition is). Landman (2004) explicitly pursues the goal of a unified theory of the semantics of measure phrases and bare and modified numerals. A numeral may be combined with what Landman calls a ‘numerical relation’ in both of its uses in (38) and (39a), as illustrated in (40) (as well as with a ‘measure term’ described below).

- (40) a. The number of Jupiter’s moons is exactly / more than / less than four.
 b. Mary is exactly / more than / less than six feet tall.

Hackl (2000), Landman (2004), Snyder (2017) and others claim, essentially following Frege, that the meaning of the numeral is the corresponding number, as shown in (41a), but that that numeral may be combined with a numerical relation which, when null, is interpreted as ‘exactly’, and with a ‘measure term’ like *feet* or *pounds* which, when null, is interpreted as cardinality. Consequently, *four* has the basic use in (41a) but, combined with a null numerical relation and a null measure term, the derived use in (41b). An overt numerical relation like *less than* and an overt measure term like *pounds* (which denotes a function μ_{lbs} that maps an individual to its measure on a scale of pounds) yields the meaning specified in (41c).

- (41) a. $\llbracket \text{four} \rrbracket = 4$
 b. $\llbracket \emptyset \text{ four } \emptyset \rrbracket = \{x \mid |x| = 4\}$
 c. $\llbracket \text{less than four pounds} \rrbracket = \{x \mid \mu_{\text{lbs}}(x) < 4\}$

The numeral in (38), then, functions as a number, or degree specification in (41a), while the numeral in e.g. *four moons* functions syntactically as an adjective with the meaning of the measure phrase in (41b), and is interpreted intersectively with the modified noun (*four moons* describes a plurality of moons with cardinality ‘four’). The meaning of the measure phrase is derived by special composition rules for Landman, which Snyder reformulates as type lifts.

Kennedy (2013, 2015) pursues a quantificational analysis of the meaning of both bare and modified numerals, where they denote properties of sets of degrees, giving them the same logical degree order as degree quantifiers like *-er*. *Four* and *more than four* hold when the maximal degree in their degree set argument is equal to or above four respectively, as spelled out in (42).

- (42) a. $\llbracket \text{four} \rrbracket = \lambda P_{\langle d, t \rangle} . \max\{n \mid P(n)\} = 4$
 b. $\llbracket \text{more than four} \rrbracket = \lambda P_{\langle d, t \rangle} . \max\{n \mid P(n)\} > 4$

Kennedy (2015) suggests that the singular term use of numerals is derived from the degree quantifier in (42a), rather than representing the basic use (see also Hofweber 2005 and Ionin & Matushansky 2006, among others, for variations on this theme). He points out that feeding *four* as defined in (42a) a value for P meaning ‘be equal to d ’ and abstracting over d derives the degree-property of being equal to four. This is accomplished by the applying Partee’s (1987) BE operator, defined in (43a), to the meaning in (42a). Applying the IOTA operator defined in (43b) to the result derives a singular term meaning ‘the number which is equal to four’ (in this case). So we are ‘back’ at four.

- (43) a. $BE = \lambda Q_{\langle \alpha, t \rangle, t} \lambda x_\alpha \cdot Q(\lambda y_\alpha \cdot y = x)$
 b. $IOTA = \lambda P_{\langle \alpha, t \rangle} \cdot \iota x_\alpha [P(x)]$

Hence, the meaning of the numeral is considered basic in Landman, Snyder, and Scontras’ accounts (as well as Kennedy’s earlier 2013 account) and the predicative use derived, while in Kennedy’s (2015) account the degree-quantifier use is basic and the singular term use is derived.

Both of these approaches to the meaning of numerals (and measure phrases in general) remove numerals from the pantheon of generalized quantifiers understood as relations between sets of individuals, just as Hackl’s analysis undermines the view of *most* as a relation between sets of individuals. The issue of whether Landman and Kennedy’s meanings for numerals are permutation invariant, and therefore qualify as quantifiers in a logical sense, is not completely transparent. Landman mentions that numerals as adjectival modifiers obey a notion of ‘quantitativity’ he describes informally. The idea is, if a plural individual meets the description *four Canadians*, and we replace one of the atoms in that plurality with another, the result might no longer meet the description *Canadians*, but it will still meet the description *four*. This makes *four* ‘quantitative’ and *Canadians* not, as desired. One way of ensuring this result is to restrict the permutations we use to those that preserve the part structure of plural individuals (or mass nouns), as van Benthem (1984, 1986) suggests. A part-structure preserving permutation ϕ is a mapping from a universe to a universe of equal cardinality that respects the condition in (44).

- (44) $\forall x, y, z \in U [x = y \oplus z \leftrightarrow \phi(x) = \phi(y) \oplus \phi(z)]$

With respect to such a condition, the truth of *Canadian*(x) does not guarantee the truth of *Canadian*($\phi(x)$) but the truth of *two*(x) still guarantees the truth of *two*($\phi(x)$), since if x consists of two elements y and z , $\phi(x)$ must consist of the images of those two elements and no others.

Recall that we concluded above that the comparative is permutation invariant (the same reasoning applies to equatives). If the degree set A is a subset of the degree set B , then any mapping of these degrees to other elements will ensure that A_ϕ is a subset of B_ϕ because the set B_ϕ is by definition just the set $\{\phi(x_1), \phi(x_2), \dots, \phi(x_n)\}$ for each x_i member of B . If all the members of A are in B then they all have images in B_ϕ . But Kennedy's definition for his numeral degree set modifiers does not involve any quantitative relation like 'subset of'. Rather, it attributes a name (a particular numeral) to the maximal degree in a degree set. The sentence *Jupiter has four moons* denotes the formula in (45) on Kennedy's analysis.

$$(45) \max\{n \mid \exists x \text{ have}(\text{Jupiter}, x) \ \& \ \text{moons}(x) \ \& \ |x|=n\} = 4$$

Suppose our permutation maps all the values for n , to, say, turtles. Then, even if we superimpose the degree order inherent in the number scale onto the turtles, so that each turtle bears a scalar relation to the other turtles reflecting the original ordering of degrees, the maximal element in this set is a turtle, not a number like *four*. But the quantifier *four* equates the maximal element in its degree set argument to the number *four*. If its argument A is a set of degrees but A_ϕ is set of turtles, then the truth of the degree quantifier *four* is not preserved across the the permutation. This situation is similar to lifts of proper names discussed by Montague (1973) mentioned in section 2. It is possible to construe a name like *Mary* as a generalized quantifier—a property of sets that is true of all the sets that contain Mary, i.e. all the properties Mary has. But this quantifier is not permutation invariant because Mary's being in a set A does not guarantee that she is in A_ϕ . Likewise for the definition of *four* in (42a) because it makes reference to a particular individual (in this case the number 'four'), like names do.

One response to this dilemma is to separate the equivalence schema from the specification of number, as Landman (2004), Scontras (2014) and Snyder (2017) do. Staying with Kennedy's nomenclature, the modifier *four* defined in (42a) actually consists of the two components in (46): the degree-denoting number four itself, and a quantificational relation MAX that combines with a number and a degree set and equates the maximal member of the degree set with the number. The number in (46a) is not permutation invariant because it names a thing, nor is the quantificational derivative in (42a) because it makes reference to the number. But the quantificational schema in (46b) that derives the degree quantifier in (42a) from the number in (46a) is itself permutation invariant. If a set A contains numbers, and these numbers are mapped to turtles preserving the scalar order between them, then the maximal turtle in A_ϕ is

the image of the maximal number in A . The relation in (46b) therefore holds across permutations of the universe, even if the number in (46a) and its quantificational derivative in (42a) do not.

- (46) a. $\llbracket \text{four} \rrbracket = 4$
 b. $\llbracket \text{MAX}(n)(A_{(d,t)}) \rrbracket = \max\{n' \mid n' \in A\} = n$

Van Benthem (1986: chapter 3) points out that the relations Partee (1987) observes between individuals, sets and quantifiers are preserved under permutation. To return to the example of Montague's lift of individuals to quantifiers, although neither the name *Mary* nor the set of properites $\lambda P.P(\text{Mary})$ are permutation invariant, the schema that maps the first to the second, namely $\lambda x \lambda P.P(x)$ is permutation invariant. This mapping bears a resemblance to the relation defined in (46b). This is significant because Landman's (2004), Kennedy's (2015) and Snyder's (2017) systems relating the various uses of numerals to one another (on which (46b) is based and which are more numerous than I have discussed here) are based on Partee's system of type changing rules. This approach, one might say, identifies the logical core of the way names for quantities are put to use in natural language, and identifies extensive similarities with rules that operate in the domain of individuals.

Just as the analyses above undermine the view of *most* and numerals as relations between sets of individuals, there is evidence that *some* has a use parallel to that of numerals. *Some* may be used as what Anderson (this volume) refers to as an 'indeterminate numeral' in expressions like *twenty-some people*. He claims that *some* in this use functions as an object language variable over numerals, which in combination with focus generates a set of alternative expressions in which *some* is replaced by a numeral. With the assertion *twenty-some people arrived*, the speaker expresses his or her ignorance about the value of *some*, but, accordingly, their certainty that the number of people who came lies between twenty and thirty. It is unclear how directly this use of *some* relates to its use as a determiner, but it implicates that *some* might not be safe from the encroachment of degree semantic analyses on the territory of generalized quantifiers as described here.

This leaves *every* as a clear case of a generalized quantifier in Mostowski's sense, and its status as a relation between sets of individuals is supported by the fact that it combines with a singular noun, as one would expect given that singular nouns denote sets of atomic individuals, as opposed to plurals, which denote sets of pluralities. Arguably *every*'s discourse-linked counterpart *each* and their complement *no* (when it combines with a singular noun as in *No Canadian is intrepid*) are also a relations between sets. It is somewhat anti-

climactic that *every* and *no* can be expressed as first order predicate logical operators, the latter in the form of negation of existence. Though other generalized quantifiers in the traditional sense may exist, it is clear that much of the work for which generalized quantifier theory was originally envisioned is in fact done by degree quantifiers (or measure phrases as Landman and others formulate them). Where exactly the boundary lies between generalized quantifier theory and degree semantics remains the subject matter of current developments in semantics. As the discussion above shows, this enterprise is intimately related to the question of how quantificational terms are composed from their morphological component parts, both synchronically and diachronically. See Szabolcsi (2010) for a recent overview of advances in this area.

The contributions to this volume address this subject matter in diverse ways. I have hinted above at the kinds of contributions that some of them make. Below, I discuss each of them in turn and clarify how they relate to the issues in individual and degree quantification discussed above.

4 Contributions

As mentioned above, Curt Anderson's "Indeterminate Numerals and their Alternatives" deals with the use of English *some* as a numeral. Strawson (1974) points out that after hearing an utterance like *Some cabinet minister has been shot!*, it would be inappropriate to ask *Who?*, since the statement implies the speaker does not know who the cabinet minister in question is. Anderson points out that this ignorance implicature carries over to the use of *some* in numeral expressions like *twenty-some*, where *some* stands in for a number. This suggests that the same *some* is being used in both cases, in the latter case clearly in the syntactic distribution of a number. Anderson proposes a compositional analysis of numerals along the lines of Landman's analysis described above, in which the numeral itself is inserted into a quantificational schema that in turn has the noun as argument. The fact that complex numerals can be manipulated by compositional processes (replacement of a number by *some* in complex numerals like *twenty-some*) supports the view that complex numerals are compositionally derived. The ignorance implicature is modeled as competition between possible values for the position held by *some*.

Barbara Tomaszewicz-Özakin's "The Semantics of Superlative *Est*" discusses the semantic type of the superlative morpheme, and presents evidence for some flexibility in its argument structure. Just as the meaning of the comparative is compatible with different argument structures (the 3-place phrasal comparative in (16) and the 2-place clausal comparative in (19)), she shows

that the denotation for superlative *-est* in (32) is in complementary distribution with another denotation that relates two degree set properties, i.e., that is missing the ‘external’ individual argument (x in (32)). Heim (1999) explored the idea that the “3-place” (because it has three arguments) denotation for *-est* described in footnote 4 above could be reduced to a two-place denotation that relates two degree sets. On the two-place definition, which bears a strong resemblance to the comparative defined in (19), the external individual argument is abstracted over an element in the scope of *-est* through association with focus. Romero (2012) claims modal superlative constructions (e.g. *the most candy possible*) implicate the existence of 2-place *-est*. In the present volume, Tomaszewicz-Özakın presents a series of new facts from Polish and Turkish that illustrate readings of the superlative not available in English, and shows that these readings are only derivable with 2-place *-est*. But she also shows that, contra Heim (1999), 2-place *-est* does not associate with focus, and certain constructions only admit 3-place *-est*. Consequently, the two semantic types for the superlative exist side by side, and their distribution is governed by the requirements of the particular context they occur in.

In similar vein, Elizabeth Bogal-Allbritten and Elizabeth Coppock describe the relation between the comparative and superlative in Navajo in “Quantification, Degrees and *Beyond* in Navajo”. First they point out that syntactically speaking, Navajo is like Hindi as described by Bhatt & Takahashi (2011) in only admitting phrasal comparatives, signalled in Navajo by the presence of the standard marker *lááh*, which functions otherwise as the preposition meaning *beyond*. Superlatives are derived by attaching the ‘unspecified object’ prefix *’a-* to the standard marker *lááh*, which Bogal-Allbritten and Coppock analyze as an operator that introduces existential quantification over an argument position. They then show that combining the standard semantic analysis for the phrasal comparative with existential quantification fails in and of itself to derive the meaning of the superlative. But the combination of the degree quantifier analysis of the comparative usually associated with clausal comparatives with existential quantification introduced by *’a-* correctly generates the meaning of the superlative. This means that the semantic type of the comparative (whether it relates degree sets or individuals) is not strictly determined by its syntactic combinatorial status (phrasal or clausal). The account also represents a concrete analysis of a case where the superlative is morphosyntactically derived from the comparative, lending support to the idea that superlative meaning is derived from comparative meaning, a notion that cross-linguistic morphological evidence also lends some credence to (Bobaljik 2012). Bogal-Allbritten and Coppock also point out the significance of their results in light of the fact that Navajo lacks quantificational determiners. While one explanation for this fact

might be that Navajo is a 'first order language', the existence of second order degree quantifiers in the comparative construction militates against this possibility.

In "Separate but Equal: A Typology of Equative Constructions", Jessica Rett discusses parallels between comparative and equative constructions in cross-linguistic perspective, and finds that just as some languages have comparative constructions that are not quantificational, non-quantificational equative constructions are also attested. She develops semantic tests for determining whether an equative construction involves degree quantification or not independently of its morphosyntactic form, and finds that all those equative constructions that do not display an overt marker associated with the degree predicate (corresponding to the first *as* in English *as tall as*) are non-quantificational. These are referred to as 'implicit' equatives. But moreover, a bifurcation is found among 'explicit' equatives, that display an overt equative marker, between those whose marker functions otherwise as resultative or sufficientive morpheme (analogous to English *so*, related historically to *as*) and those whose marker functions otherwise as a degree demonstrative (analogous to English *that much*). The former is compatible with a factor modifier (*twice as much as*). The latter is not, but is otherwise similar to the former; in contrast to equatives without an overt marker, both classes of marked equative are non-evaluative (they do not entail that the degree property holds to a high degree; i.e., *Mary is as tall as John* does not mean that she is tall) and both classes have an 'at least' reading (*Mary is as tall as John* is compatible with her being taller than him). Rett develops a semantic analysis of both the non-quantificational implicit equatives, which involve a covert similarity relation, as well as for the equatives with a degree demonstrative marker, which involve direct reference to a particular degree. Specifically, the standard clause is type lowered to denote the maximum degree in the underlying degree set the standard clause denotes, which is then identified with the degree the demonstrative denotes in the main clause. Equative constructions, then, can be constructed through reference to degrees, rather than quantification over degrees.

In "Compounded Scales", Alan Bale takes up the issue of what exactly degrees are, and develops an approach to the comparative that bears a stronger resemblance to individual quantification than degree quantification. This approach pursues a possibility described by Cresswell (1976) that the values for *u* in the pairing $\langle u, > \rangle$ that in his system constitute a degree (see (15)) are not abstract coordinates but actual individuals. A scale of degrees of height determines a scale of individuals having those (maximal) degrees of height and vice versa. But Bale shows the two notions of scalarity (as rankings of individuals or of degrees) are not equally linguistically useful. Taking *-er* to relate

individuals rather than degree sets presents a simple solution to the otherwise recalcitrant problem of why gradable predicates can be coordinated. On the degree quantifier view, *Seymour is more handsome and talented than Patrick is* is predicted to mean that Seymour has exactly the same degree of handsomeness and talentedness, and that this exceeds Patrick's degree of handsomeness and talentedness. But the sentence does not in fact require Seymour to have the same degree of handsomeness as talentedness, nor would this normally even be possible; handsomeness and talentedness correspond to different degree scales and do not intersect at all. But if, as Bale recommends, we understand *handsome* and *talented* to denote relations between individuals, so that Seymour bears the *handsome* relation to everyone he is more handsome than, then he can very well bear the conjunctive predicate *handsome and talented* to some other individual under the standard notion of what *and* means. In Bale's system, a 'degree' of some gradable relation is the set of individuals that bear that relation to all the same individuals, e.g., those that are identically handsome in the case of the gradable relation *handsome*. The comparative morpheme *-er* has a denotation similar to the phrasal comparative defined in (16) on this view (where in Bale's system the compared-to degree is derived from the *than*-clause by maximalization). Since each of the individuals in such an equivalence class can be replaced with any of the others preserving truth, this definition of *-er* (as relations between equivalence classes of individuals) is permutation invariant as long as the permutation respects these equivalence classes. Here as in the case of the (3-place) superlative discussed above, which also has an individual argument, the permutations we take into account in testing for invariance must preserve the various gradable relations in the model in order for the comparative to qualify as quantificational in this sense.

In a similar vein, Roger Schwarzschild develops an analysis in "From Possible Individuals to Scalar Segments" in which comparative constructions rank individuals rather than relating them to points on a scale. Developing Cresswell's suggestion mentioned above, he defines a degree as a set of pairs of individuals and possible worlds—'possible individuals'. Gradable predicates denote relations between individuals and degrees in this sense. Specifically, a gradable predicate relates an individual x to the set of possible individuals that x is greater than or equal to on the dimension in question. The central concept at work in comparative constructions is that of a 'segment'—a triple consisting of a starting degree, an ending degree, and a measure function μ_σ (where σ indexes the segment in question) that maps individuals to degrees, where degrees, again, are sets of possible individuals. A sentence like *Anu is taller than Raj* describes a segment whose measure is height, whose starting degree is $\mu_\sigma(\text{Raj})$ and whose ending degree is $\mu_\sigma(\text{Anu})$. Differentials are descriptions

of such segments that specify the difference between the start and end points. The chapter explores various ways of morphologically dividing up the work of specifying the properties of segments. Ultimately, the English morpheme *-er* is argued to set the endpoint of a segment, and *-er* and the differential (when present) are argued to be present in both the matrix clause and the standard (*than*) clause in comparatives. The main work of comparison is done by *than*, which combines with the matrix and standard clauses to derive a description of a segment whose endpoint is specified by the matrix clause and whose starting point is specified by the standard clause. Because it puts a copy of the differential phrase (when one occurs) in both the main and standard clauses, this analysis accommodates cases in which a differential occurs in the scope of a world-introducing predicate in the matrix clause that is in turn in the scope of the comparative. For example, *Jack expects the engine to be one boxcar longer than Jill does* can be true even if Jack and Jill are not sure about the length of a boxcar, a situation that is paradoxical on other analyses of the comparative. This analysis also goes some way toward explaining why languages draw heavily on vocabulary associated with movement in space or time in comparative constructions, as in the Navajo construction discussed in Bogal-Allbritten and Coppock's contribution. There, the standard phrase is introduced by the spatial preposition *lááh* 'beyond', which specifies the starting point of a segment in Schwarzschild's analysis of the comparative, just as it specifies the starting point of a vector in its spatial use.

In "Measuring Cardinalities: Evidence from Differential Comparatives in French," Rajesh Bhatt and Vincent Homer discuss a curious restriction on differentials in French that relates to the proper analysis of quantity comparatives. In French, quality comparatives with differentials bear a strong resemblance to English, but differentials in quantity comparatives fall into two groups. Differentials that are themselves degree quantifiers like *beaucoup* 'a lot' pattern like English (*beaucoup plus de livres que* 'many more [of] books than'), but cardinal differentials cannot appear pre-nominally like other differentials do in French (**trois livres plus que* corresponding literally to 'three books more than'). It is possible to rescue this structure by inserting *de* 'of' before *plus*, as in *trois livres de plus que*, literally 'three books of more than'. However, Bhatt and Homer show that this has an entirely different structure from *beaucoup plus que* 'many more than', namely one in which the differential *trois livres* is the head of the nominal construction, and *de plus* a modifier of this noun. They postulate that *plus* has the same interpretation as elsewhere, it is a degree quantifier that relates two degree sets. In a sentence like *Marie a lu trois livres de plus que Jean* 'Marie has read three books [of] more than John', quantifier raising of the whole object noun phrase derives an abstract over things Marie

read. As such, this is not an appropriate argument for *plus*, which, like *-er*, relates degree sets. However, borrowing from Grosu & Landman (1998), they claim that an operator occurs in object position that converts the degree argument into the corresponding amount of ‘stuff’, which must be books in this case to be commensurate with the book restriction on *plus*. This allows them to treat the abstraction over books that Marie read instead as an abstraction over degrees corresponding to the amounts of books Marie read, which provides an appropriate argument for *plus*. This mechanism allows for degree argument abstraction over individual argument slots in the syntax, which may play a role in quantity comparatives and superlatives more generally. It also shows, like the preceding papers, that the same underlying semantic relation between degree sets may manifest itself in a flexible variety of syntactic structures.

Haley Farkas and Alexis Wellwood experimentally investigate the interpretation of adverbial *more* in “Quantifying Events and Activities”. A sentence like *Ann jumped more than Betty* does not explicitly specify the dimension on which *more* should be interpreted. It could mean she jumped higher, more times, or for a longer duration, raising the question of how the dimension of degree quantification is determined when it is underspecified by the syntactic context. In a series of experiments, Farkas and Wellwood investigate the role of the aspectual character of the modified verb and the role of aspects of the visual scene so described in making this determination. They ask participants (in separate blocks) whether one object in a visual scene they viewed moved or jumped more than another object, and compare their responses to responses to questions that ask directly whether the first object moved or jumped more times, longer, or higher than the second. Participants tend to evaluate *jump more* as meaning *more times* while *move more* can mean *more times* or *higher*, but still resists a *longer duration* interpretation. Two additional experiments show that altering aspects of the visual scene does not significantly change the responses recorded in the earlier experiments (insofar as the description *jump/move* is still appropriate to the scene). The fact that *more* is interpreted equally readily as degree quantification over times and height means that participants do not simply display a ‘quantity over quality’ bias in the interpretation of the comparative. The dimension of measurement is analysed by Schwarzschild (this volume) as the sole contribution of the gradable predicate in comparative constructions. In cases where the predicate modified by *more* is not itself gradable, Farkas and Wellwood show that the choice of dimension of gradability is nonetheless more heavily influenced by the meaning of that modifyee than by other factors. The interaction between event structure and the choice of dimension for the comparative reported here also reflects a more general interaction documented by Stassen (2006) between the morphologi-

cal components of the comparative and the vocabulary of time and space, as exemplified in Navajo as described by Bogal-Allbritten and Coppock (this volume).

The final two contributions to this volume deal with the persistently challenging problem of the interpretation of individual quantifiers within *than*-clauses. In “Split Semantics for Non-monotonic Quantifiers in *Than*-Clauses”, Linmin Zhang addresses the problem of non-monotonic cardinality quantifiers like *exactly two boys* when they occur in *than*-clauses. She begins with a non-monotonic interval theoretic analysis where *Mary is tall* denotes the minimal set containing Mary’s exact height and no other degrees. *Mary is taller than John* says that there is a positive difference between Mary’s height interval and John’s. *Mary is taller than everyone* says that there is a positive difference between Mary’s height interval and the smallest interval containing the heights of everyone else. In order to deal with *exactly two* in the *than*-clause, Zhang marries the interval theoretic approach to a dynamic semantic treatment of uniqueness based on Bumford (2017). On this approach, discourse referents are introduced for a plurality of boys and the interval including their heights as the structure is built, and at the sentence level it is asserted that there is a positive difference between Mary’s height interval and the height interval of the boys. Only at this point is the uniqueness of *exactly two* and of *than* imposed. The latter requires the interval that includes the two boys’ heights to be maximal, in this case meaning it extends from the lower bound of Mary’s height interval downward indefinitely, and the former requires that the boys that have their heights in this interval be exactly two in number. This excludes the possibility that there is any third boy who is shorter than Mary. All the other boys are taller than Mary, if there are any. This analysis therefore captures an enigmatic interaction between quantification and degree, that allows non-monotonic quantifiers to be interpreted within *than*-clauses without falling victim to the false predictions that standard monotonic degree semantic analyses of the comparative make in such cases.

Nicholas Fleisher presents another strategy for dealing with quantifiers in *than*-clauses in “Nominal Quantifiers in *Than*-Clauses and Degree Questions”. He capitalizes on parallels between the behavior of quantifiers in *than*-clauses and in embedded questions, and analyzes *than*-clauses accordingly along the lines of degree questions (that ask *how much*, *how many*, etc.). Just as a naive degree semantic account of *John is taller than every girl* expects it to mean that he is taller than the shortest girl, as discussed in section 3, the embedded question *John knows how tall every girl is* is expected to mean that he knows the height of the shortest girl. Chierchia (1992) and others argue that the attested reading of embedded questions with quantifiers is a pair-list reading (a pairing

of girls and their heights representing what John knows). Fleisher shows how pair list readings can be generated for *than* clauses containing quantifiers on the model of Chierchia's approach to embedded questions in combination with Nouwen and Dotlačil's (2017) claim that the quantifier has wide scope within the *than*-clause. He also shows that of two other readings for embedded questions, one, the single-point reading that presupposes, in the example above, that all the girls have the same height, carries over to *than*-clauses, though the reading is inconspicuous for pragmatic reasons. Another reading of embedded questions, the functional reading that licenses the continuation 'taller than her mother' to the embedded question cited above, is not available for *than*-clauses. But Fleisher attributes this to independent differences between embedded questions and *than* clauses. Unlike embedded degree questions, *than* clauses involve an operator that converts the most informative answer to the question to the degree that makes it true, for the purposes of comparison with the matrix clause denotation. The functional reading is not type-compatible with this conversion process. In this manner, Fleisher collapses the *than*-clause enigma with existing accounts of the interpretation of embedded degree questions, a unification that various empirical parallels warrant.

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