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6. THE ROLE OF CONTINUOUS ASSESSMENT AND EFFECTIVE TEACHER RESPONSE IN ENGAGING ALL STUDENTS

INTRODUCTION

In this chapter, we report on three practices that, together, reduce marginalization of students from effective learning in mathematics. We have observed that the combined use of continuous formative assessment, responsive teaching, and effective patterns of variation form a powerful way of engaging and sustaining mathematics learning for students who might otherwise be marginalized by being left behind or not being appropriately challenged. Our observational categories draw from work in formative assessment and variation theory, our own approach to mastery learning, and iterations of testing and refinement in light of our own classroom observations. We begin by situating the practical and theoretical perspectives of our work with respect to mastery learning and variation theory, and then further explain the relevance of our work in preventing marginalization of learners. From there, we describe our work in classrooms and its role in the development of an observation protocol based on the categories we describe. Finally, we offer four classroom episodes that highlight the significance of these categories.

Decades of research and practice of *mastery learning* (Bloom, 1968; Guskey, 2010) have confirmed the importance of assigning work to students only when they are ready to engage in such work. Identifying the time a student is ready to move forward is particularly relevant for learners disadvantaged by issues such as ability, language, culture, and socio-economic status. We believe teaching practices that neglect mastery learning marginalize students who, for whatever reason, are not able to engage productively in the classroom discourse. We propose that *continuous formative assessment* followed by *appropriate teacher responses* during instruction supports all students' *engagement* in classroom activities and plays a key role in developing equitable teaching practices. Although formative assessment strategies have been extensively suggested by recent literature (e.g., Box, Skoog, & Dabbs, 2015; Chappuis, 2015; Heidi & Cizek, 2010; Wiliam, 2011; William & Leahy, 2015), there is little advice for the teacher on how to respond in the moment to emergent situations in the classroom. We argue that such responses can be informed by Marton's (2015) *variation theory of learning*, addressing not only those students

who struggle to understand a particular mathematical instruction, concept, or procedure, but also, students who quickly master the targeted content or goal of a lesson.

In this chapter we present some results from research on an intervention aimed at improving numeracy at the elementary level in Canada. The research is part of a partnership among a major school district (Calgary Separate School District), a charity that develops mathematics curricular materials (JUMP Math), and the Werklund School of Education of the University of Calgary. We have observed, through longitudinal data, a steady improvement of performance in mathematics. However, not all the groups showed similar improvement, and teaching practices varied from one classroom to another.

We have identified three teaching practices that seem to be key in allowing students to engage meaningfully in the mathematical discourse in the classroom, namely: continuous assessment of all students during class, particular forms of teacher responsiveness, and use of effective variation to draw attention to critical content features. We have adapted previous work on mastery learning by (a) placing a stronger emphasis on prevention rather than remediation, (b) stressing the importance of moment-by-moment assessment to all students, and (c) engaging all students in the same type of activities instead of streaming students for remediation or enrichment. These practices are used in this chapter to describe how teachers can support all students' learning; we argue that when one or more practice falters, students are marginalized through lack of opportunity to engage meaningfully in mathematics learning activities in class.

MARGINALIZATION AND MATHEMATICS EDUCATION

Mathematics ability, or lack thereof, has been identified as a marginalizing factor impacting people's lives. A recent report from the Organization for Economic Co-operation and Development (OECD, 2016) acknowledged that "Numeracy skills are used daily in many jobs and are important for a wide range of outcomes in adult life, from successful employment to good health and civic participation" (p. 36). In many cases, mathematics is a filter for further education; students need to complete high school programs and to pass admission tests that include mathematics to be accepted in postsecondary programs. The OECD report also concluded: "[M]athematics education often reinforces, rather than moderates, inequalities in education" (p. 3). In this sense, poor quality instruction in mathematics from the early years perpetuates and may even foster social inequity.

The OECD (2016) report has identified persistent problems in mathematics education; however, the proposed solutions seem very general with no attention to particular mathematical content or to particular mathematical knowledge required for teaching. One of the problems is that K-12 mathematics instruction seems to be differentiated with regard to socio-economical status and initial mathematical ability, as reflected in identified differences across schools:

[T]eachers use cognitive-activation strategies to deepen the curriculum content and support the development of problem-solving abilities among students in advantaged schools. By contrast, in disadvantaged schools, it appears that there might be a price to pay for using strategies that emphasize thinking and reasoning for an extended time: less material is covered. (p. 113)

This statement, however, seems to place problem solving and content coverage at odds with one another for those disadvantaged schools. The proposed solutions are based on findings from the Programme for International Student Assessment (PISA), which advises teachers to: encourage students to work in small groups; provide extra help to students who need it; reduce the mismatch between taught content and assessment; and promote the use computers in the mathematics classroom. These suggestions do not consider a more nuanced mathematical knowledge for teaching, which has been studied for decades by the mathematics education research community (see Da Ponte & Chapman, 2016).

In this chapter, we not only consider general mathematics teaching practices, but also their relation to a nuanced knowledge of critical features for learning particular mathematical ideas (Marton, 2015). These features inform both assessment and instruction. In our experience, learning gaps often get interpreted as learning disabilities, and boredom may be interpreted as lack of motivation. Both interpretations may contribute to marginalizing students from learning mathematics and, therefore, from life opportunities. The teaching practices we focus on are intended to address both learning gaps and lack of motivation.

Many issues of inequity and social justice regarding marginalized populations go well beyond the scope of this chapter. However, regardless of the factors impacting equity or of the approach for breaking barriers impacting marginalized students, we stress that teachers and schools must consider teaching practices and teachers' knowledge that support the learning of all students in the classroom. While there is an obvious emphasis on disadvantaged students in terms of factors such as socioeconomic status and language, we also consider practices that marginalize students who meet basic expectations but, with better instruction, could develop deeper mathematical understanding. Students who have mastered expected learning outcomes may be marginalized from further learning if they are not provided with activities or tasks that extend their mathematical ability or knowledge.

THEORETICAL BACKGROUNDS

The teaching practices that we attended to in the intervention were informed by the extensive research on mastery learning and by the variation theory of learning: While the former is an instructional approach, the later is a theory highlighting necessary conditions for learning that can inform teaching.

The variation theory of learning prompts attention to key features necessary for understanding particular ideas (in our case, mathematical ideas), and to developing patterns of variation that draw attention to these features (Marton, 2015). According to this theory, we learn through experiencing patterns of variance against a background of relative invariance. For instance, a student can learn what a square is by contrasting squares with other shapes, each of which differ from a square in a particular way (e.g., more than 4 sides, unequal sides, angles that are not right). Such contrast would prompt attention to the critical features to be discerned in order to identify a square, which is what Marton described as the *intended object of learning*. This student may generalize her understanding of a square by considering squares in different positions (shape is invariant and position varies) or with different patterns. In other words, the learner first discerns the critical features of a square, and then discerns non-essential features. In both cases, certain patterns of variation against a background of invariance allow the learner to focus attention on critical features of the intended object of learning. While this theory has similarities to the development of concepts described by Bruner (1960), it is not inherently constructivist. Nonetheless, Runesson (2005) showed how using variation theory as an analytical framework may complement constructivist or social constructivist analyses, as it draws attention to features not considered by either.

Mastery learning (Bloom, 1968; Bloom, Hastings, & Madaus, 1971; Guskey, 2007, 2010) emphasizes formative assessment, remediation, and enrichment. In the intervention described in this chapter, we combined these ideas with instructional practices that emphasize the use of effective patterns of variation to draw attention to the critical features of clearly-defined objects of learning. We will now elaborate the way in which we bring these ideas together.

Guskey (2010) suggested that formative assessments should vary in frequency from one to several weeks. In his view, a key purpose of these assessments is to identify students who have already mastered the learning goals and those who need *remediation*. Teachers may then prepare “*enrichment activities that provide valuable, challenging, and rewarding learning experiences for learners who have mastered the material and do not need corrective instruction*” (p. 3). These activities are often selected by the students and might involve projects or reports, games, or complex problem-solving tasks (Guskey, 2007). We concur with Guskey (2010) and Bloom (1968) that enrichment activities should be more than a way to keep students busy and thereby allow teachers time to support other students and that they should also provide opportunities for deepening students’ learning. However, we further emphasize the role of using of effective patterns of variation and continuous assessment in preventing the need for excessive remediation, and we stress the importance of continuous extension for *all* students.

The focus on remediation initiated by mastery learning is evident in the extensive literature on formative assessment (e.g., Box, Skoog, & Dabbs, 2015;

Chappuis, 2015; Heidi & Cizek, 2010; Stiggins, Arter, Chappuis, & Chappuis, 2004; Wiliam, 2011; William & Leahy, 2015), which prompt to remediation. Further, while many scholars have agreed that formative assessment should happen moment by moment, the advice for teachers is often expressed in terms of general strategies, such as re-explaining or re-teaching the lesson. Little is said about how the explanation or lesson could be adjusted to better meet students' needs.

The approach to formative assessment we describe here differs from mastery learning as it has been commonly reported in the literature: We have stressed the need to assess not only *regularly* at the end or in the middle of a unit of content, as suggested in mastery learning, but also *continuously* during class time with a focus on *prevention* rather than *remediation*. The idea is that teachers fine-tune instruction moment by moment, including posing extra activities for some students and addressing students who require support. Similar to the enrichment activities in mastery learning, the curricular material used by teachers in the intervention advises teachers to “be ready to write bonus questions on the board from time to time during the lesson for students who finish their quizzes or tasks earlier” (Mighton, Sabourin, & Klebanov, 2010, p. A-8). More specifically, it suggests that teachers may: use larger numbers; introduce new terms or elements; ask students to correct mistakes; ask students to complete missing terms in a sequence; vary the task or the problem slightly; look for applications of the concept; and ask students to find and describe patterns. Consistent with Mighton (2007), teachers in the intervention were also encouraged to prepare extra material for *all* students, including those who initially performed lower in class. These extra activities should be tied to the common task that everyone is doing in the lesson. Further, proposed activities are often small variations of the tasks in which all students engage in class, but with the potential to prompt emergent insights regarding the broader mathematical ideas (cf. Aljarrah, Preciado Babb, Metz, Sabbaghan, Pinchbeck, & Davis, 2016). This offers another contrast with many of the enrichment activities described with respect to mastery learning (Guskey, 2007), as these typically extend beyond the learning goals of the lesson.

Variation theory (Marton, 2015) has the potential to inform both initial instruction and teachers' responses to student feedback, supporting both students who need it and challenging those who have met the expectation of a lesson or a part of a lesson. In this intervention, we have used strategies for continuous assessment during class that are similar to Wiliam's (2011) all-student response systems and Wylie and Wiliam's (2007) hinge questions; that is, questions are often framed such that students may answer them in less than a minute, allowing the teacher to make informed “on-the-fly” decisions. These decisions can be informed by critical features of the intended object of learning, as described by Marton (2015). For instance, Sabbaghan, Preciado Babb, Metz, and Davis (2015) proposed a type of *micro-scaffolding* informed by the variation theory of learning: After noticing that not all students are providing the expected answers in short questions, the teacher presents an easier task to the students—one that everyone can understand—and then

re-builds the concept through a series of tasks informed by structured variation (Metz, Sabbaghan, Preciado Babb, & Davis, 2015). Students who more quickly demonstrate an understanding of a particular aspect of the lesson are offered direct extensions of completed tasks; these consist of small but thoughtful variations of the completed task.

Our focus on teaching practices includes how teachers use educational resources in the classroom. The research in this intervention also focused on teacher and student interactions with provided resources, as well as consideration of the possibilities that such resources offer for students' learning opportunities.

DATA AND RESEARCH METHOD

The intervention was a design-based research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) focused on teachers' knowledge, the relationship with a particular curriculum material, and the impact on students' learning. Consequently, results from the ongoing research have informed the direction of the intervention since its beginning in 2012. The study initially involved one elementary school (K to 6 with around 150 students) with a long history of poor performance in mathematics, high diversity in terms of language and ethnicity, and common issues related to a low socio-economic status. We will refer to this school as School 1. Another elementary school (K to 5 with a similar number of students) became involved in the research in 2014. We refer to this school as School 2. Both schools received JUMP Math materials including assessment-and-practice books for each student, a teacher's guide, and pre-designed Smart-Board slides for each teacher. Additionally, mini-whiteboards were provided to every student as a means to assess their understanding during instruction (Wiliam, 2011). The instructional practices suggested in the lesson plans include direct instruction, problem solving, guided discovery, and both individual and team work.

Teachers were expected to adopt and follow the curricular materials from JUMP Math, participate in targeted professional development, and attend observations from the research team. Professional development for teachers was planned in conjunction with the research team, a representative from JUMP Math, and a representative from the school district.

The data used for the findings reported in this chapter include weekly classroom observations, video recordings of lessons, and year-end student and teacher interviews. The research focused strongly on School 1. A member of the research team observed each group on a weekly basis, providing feedback to teachers. Video-recordings were scheduled once a month. However, for a variety of reasons, including medical and maternity leaves and the reluctance of some teachers to be video-taped, it was not possible to record all the videos according to plan. Table 1 shows the number of students and the number of videos recorded sorted by grade level over two years at this school.

Table 1. Groups, students and number of videos for school 1

2014–2015			2015–2016		
<i>Groups</i>	<i>Students</i>	<i>Videos</i>	<i>Groups</i>	<i>Students</i>	<i>Videos</i>
Grade 1	19	7	Grade 1	19	7
Grade 1	20	1	Grade 1	22	9
Grade 2	25	1	Grade 2	17	9
Grade 2/3	27	0	Grade 2	17	8
Grade 4	24	8	Grade 3	27	6
Grade 5	19	7	Grade 4	18	4
Grade 6	18	1	Grade 5	24	8
			Grade 6	18	7

There were fewer video recording sessions in School 2 than in School 1. During the 2014–2015 school year, the research team only planned to video-record three lessons during the year. The data from this school is shown in Table 2.

Table 2. Groups, students and number of videos for school 2, 2014–2015

<i>Groups</i>	<i>Students</i>	<i>Videos</i>
Grade 1	20	3
Grade 2	26	3
Grade 3	8	2
Grade 3	19	3
Grade 4/5	22	3
Grade 4	21	3
Grade 5	24	3

A classroom observation protocol was initiated in 2014 with the purpose of identifying teaching practices that impact students' engagement in, and learning of, mathematics. Three team members, who observed lessons from participant teachers weekly, participated in the development of this protocol. The protocol, which was used to record written notes on each observation, initially contained more than ten categories. Based on weekly discussions throughout the 2014–2015 school year, we condensed the protocol to four categories and developed descriptors for four levels within each category. In 2015, the categories and descriptors were refined as the three researchers watched and rated the 20 video-recorded lessons from School 2. The videos were analyzed and scored on a scale of 1 to 4 for each category. No transcriptions were used for this analysis. We then checked for consistency among

members of the research team. Disagreement about ratings prompted further discussion and refinement of descriptions for each category. After two rounds of such refinement, our scores were very consistent. We describe the categories and descriptors in the next section.

While these continue to evolve as we analyze further data, the key ideas have remained stable. Here, we show how the categories may be used to identify teaching practices that foster all students’ engagement in mathematical activities and learning, thus preventing marginalization from opportunities to learn. The findings presented in this chapter were selected from the data set to showcase how particular teaching practices can either support students’ learning or marginalize students during class.

FRAMEWORK FOR CLASS OBSERVATION

The four categories that are the focus of our class observations are presented in Table 3, including an overall guiding question for each category. The first three categories correspond to the teaching practices we focus on in this chapter. Specific descriptors for the rates we assigned to each category are still under develop and are beyond the scoop of this chapter.

Table 3. Four categories for classroom observation

Category	Guiding questions
Continuous assessment	How does the teacher collect information from all students to make decisions during class?
Teacher’s responses	How does the teacher respond to the information from students?
Attention to critical features	How does the teacher prompt attention to critical features of the concepts, procedures or ideas taught in class?
Student engagement	How do students engage in the mathematical activities during class?

Continuous Assessment

Higher scores in this category were assigned when the teacher effectively used strategies for making student responses visible at several moments during class. To do so, he or she might have looked at simple responses on individual students’ mini-whiteboards or asked students to hold their hands near their chest and using their fingers to show their answers; one way or other, the teacher systematically ensured every child was checked. By contrast, lessons received a low score when the teacher failed to assess all students or failed to obtain information at key points during the lesson. A common source of low scores was when a teacher asked a question to

the group, one student gave the correct answer, and then the teacher moved on, assuming that all students had requisite understanding to continue with the lesson.

Teachers' Responses

As indicated before, there is a substantial body of literature with suggestions for formative assessment, but there is scarce indication of what to do with the information collected from these assessments. This category focuses on whether the teacher responded to student feedback. Higher scores in this category were assigned when the teacher used students' responses to guide the next steps of the lesson. This might include modifying or skipping pre-designed slides, practice pages, or parts of a lesson as well as posing further challenges or extension to students who met the goals of a lesson or part thereof. The lower scores in this categories corresponded to lessons in which teachers continued with a planned lesson without considering student feedback. Note that here we did not judge the effectiveness of the response, but whether there was a response. The effectiveness of the response is addressed in terms of effective variation in the following category.

Attention to Critical Features

This category was directly informed by Marton's (2015) variation theory and includes both the way material and activities were initially posed to students and how teachers responded to student feedback. Such responses were typically influenced both by teachers' own knowledge and by the educational material used for instruction. Descriptions of lessons scored at the higher level include attention to critical aspects of well-identified objects of learning and the use of task sequences that systematically varied one critical aspect at a time. High scores also included opportunities for students to attend simultaneously to critical features mastered prior to, or during, a particular lesson. Variations of a task or activity might be part of the planned lesson or a response to student feedback. Lower scores for this category included cases in which more than one object of learning was conflated in the lesson (or lesson segment) or in which effective patterns of variation were not used to draw attention to critical features. Questions or tasks may not have effectively built on previous ones. Support for struggling students was limited to repeating an initial explanation, while extra challenges for students who completed the work sometimes failed to consider critical aspects, thereby resulting in student frustration. Low scores for variation also occurred when students shared different strategies to solve a problem, but the teacher did little to compare and contrast them.

Student Engagement

In this category we observed students' participation in the mathematical activity of the lesson. Lessons that scored higher included cases in which most or all students

participated without prompting in nearly all parts of the lesson, and students either requested further challenges or created their own extensions for themselves or their peers. Lessons that scored lower in this category included occasions in which many students were waiting for help or extension. In some cases, large numbers of students were incapable of completing the work without individual support; in others, students engaged in activities not related to the lesson, such as reading a book or drawing at their desks.

EPISODES OF TEACHING PRACTICES AND MARGINALIZATION

We used the developed framework to analyse further data from the research. We selected four teaching episodes to showcase how different levels of attention to critical features and continuous assessment were enacted. The first two episodes serve to contrast assessment during class; both teachers offered strong patterns of variation in their teaching and responses, but a key critical feature was overlooked in the Episode 1, and weak assessment practices resulted in a lesson that was largely remedial with many students waiting a long time for the help they needed. Episodes 2 and 3 are presented to contrast teachers' responses: In both lessons, students were assessed and demonstrated early mastery of the respective topics. In Episode 2, however, the teacher prompted students to move ahead, whereas in Episode 3, the class spent an inordinate amount of time rehashing material that most found easy. Episode 4 describes a lesson where the teacher used a strong assessment and response pattern to help uncover a more effective pattern of variation than the one offered in the resource.

Episode 1: Medium Variation, Weak Assessment, Weak Response

The object of learning in this Grade 2 lesson was for students to recognize the core (i.e., the repeating part) of patterns for which the first and last terms are the same (e.g., ABA ABA); such cores were separated for particular attention due to the observed difficulty students often have in separating them from a longer sequence. In this episode, we observed that (1) the manner in which the teacher assessed made it difficult to address students' struggles at the beginning of the class and (2) attention to certain critical features of the intended mathematical object was not prompted.

The teacher began the lesson by leading a class discussion on how to identify patterns and their cores. She carefully drew attention to the meaning of patterns, cores and terms, and she used contrasting examples to illustrate a core that starts and ends with the same term:

<i>Teacher:</i>	Who can tell me what the main part of a pattern is, so we know there is a pattern out there?
<i>One student:</i>	It's the core.

Teacher: It's the core; that's right. What do the cores do so that we know that a pattern is happening?

Another student: Repeat itself.

Teacher: It has to repeat itself. If I did this, [clapping twice, putting her hands on her head and pausing]. Do you know what the core is or not? [*Sequence 1*]

Students in chorus: No.

The teacher then clapped twice, touched her head and put her hands on her knees (repeated three times), and contrasted this sequence of moves with the initial example. [*Sequence 2*]

Teacher: To have a core, the core has to repeat in order for us to know what the core looks like, correct?

This excerpt exemplifies features of variation and assessment. The teacher contrasted two sequences of moves. In Sequence 1, it was not possible to identify a pattern, because nothing repeated; Sequence 2 offered 3 repetitions of the core. The variation in these examples prompted students' attention to a critical feature of repeating patterns: They must have a core that repeats. Regarding assessment, the teacher received student feedback in the form of individual responses or from the whole group in chorus.

This response pattern was similar in other parts of the episode. At one point, the teacher called on three volunteers to generate patterns using hand signals. All of these patterns had four-term cores with different start and end terms (e.g., ABAB, AAB, ABBB). Following these, the teacher contrasted an example of a core beginning and ending with the same term with one that did not; however, when she asked whether both cores started and ended with the same term, only a few students responded. In other words, although she attempted to use contrast to highlight the critical feature of starting and ending with the same term, she did not assess whether all students actually made this discernment.

The teacher then prompted attention to three cores that started and ended with the same term. She asked individual students to identify the term that started and ended the core in each case. She then provided a non-example. Again, she asked a single student to identify whether the first and last terms were the same, and moved on once the answer was provided. Next, the teacher showed a core (with the same first and last term) and asked an advanced student to draw and repeat the core on the board by placing the terms in pre-drawn circles. Two other cores were presented, and the teacher drew attention to the matching beginning/ending terms. During this time, only those students invited to the board were assessed.

The teacher then used three connecting cubes to create a core that began and ended with the same color. She elicited what colors she would need to repeat the core and had the students chant the colour sequence. The teacher gave each student connecting cubes with which to create several copies of their own cores. They were

asked to join the cores to form a longer sequence (“core trains”), and then copy the pattern onto grid paper.

Throughout this practice segment, students worked individually and the teacher checked on their work by either walking to different tables or by sitting at a table where students came to her to show their work. Many students had difficulty creating core trains: They successfully created the first core, but began the second core with a different color cube than the color of the ending cube of the first core. For example, if the first core were red-blue-red, they would make the second one blue-red-blue, resulting in an AB pattern (red-blue, repeat). In isolation, each core appears to have just such an alternating pattern, and students had difficulty recognizing that the entire core needed to repeat (red-blue-red, repeat). Once joined, the place where one core ends and the other begins creates a spot where one colour repeats itself (ABAABA) that is not evident in the isolated cores. As a result, students perceived the pattern as an AB rather than an ABA pattern.

The teacher attempted to remediate this problem by asking students to first create three identical cores and then join them. Many students were able to create trains using these instructions. However, when they were asked to draw their trains, the previous confusion between AB and ABA patterns persisted. To remediate, the teacher asked students to dismantle their trains into core constituents and draw them individually. In some instances, students tried to reconnect the cores in an effort to make sense of why their drawings did not match their trains, but they were told to stop what they were doing, break their trains into cores, and draw them. The teacher then spent a great deal of time working with individual students in an attempt to remediate their difficulties.

From this episode, we stress that the teacher attempted to draw attention to critical features that would support students in designing patterns composed of cores with matching start and end terms. However, students did not recognize that the place where one term ends and the other begins results in a term being repeated at that point in the pattern (e.g., ABAABA; ABBAABBA). As initial feedback was largely based on individual student response, it took some time for the teacher to identify and respond to this difficulty with a pattern of variation that contrasted separated cores (ABA ABA), which can easily be perceived as a simple alternating pattern, with combined cores (ABAABA) that remove the illusion of alternating colours. Although such a contrast may have been effective earlier in the lesson, by this point it was remedial, and most of the remediation was done through one-on-one consultations with students as they worked (or waited).

Episode 2: Strong Variation, Strong Assessment, Strong Response

In this episode from a different Grade 2 classroom, we highlight a combination of patterns of variation (both in the resource and in the way the teacher emphasized important distinctions) aimed at helping students discern tens and ones in numbers to 20. We draw particular attention to the teachers’ assessment of all students *from*

the beginning of the class. In this case, she adapted her planned lesson in response to student feedback: Students seemed to understand the topic faster than expected and the teacher decided to ‘pick up the pace.’ The episode is part of a longer lesson aimed at counting up to 100.

The teacher opened the lesson by having all students count in unison to 20. She put up a slide with numbers from 21 to 29 arranged vertically, which helped draw attention to the constant two in the ten’s place and the increasing pattern in the one’s place. She asked individual students to find and show the number after 20, then 21 and 22. Building on an insight from a student, she then drew attention to the changing patterns in the one’s place. Before moving past 29, she contrasted 30 with twenty-10, thir-zero, and thirteen (in terms of how they are written as well as how they sound). The teacher asked students what came after thirty; a student said 31. She then asked what comes next, and asked students to write the answer on their mini-boards. Most wrote 32; she acknowledged this, and drew attention to the fact that the 3 for thirty stayed the same, while the 1 changed to a 2. She then allowed students to continue from 31 to 39, this time with each student writing the sequence on their mini-whiteboards. All were successful, some with a bit of help as the teacher circulated; they chanted the sequence together before she asked them what comes after 39. Here, the teacher emphasized that the three needed to go up by one and that the next number would have a “ty” on the end. One student identified 40, and all continued from 41 to 49 on their whiteboards. Again, the teacher checked in to see if students knew what came next; she emphasized the connection between “fif” and “five” and reminded them of the “ty” suffix. At this point, she had not formally assessed all students’ ability to move from a number ending in 9 to the next set of 10; however, many students were moving ahead, and some could be heard complaining that the work was too easy. She asked if they wanted to continue writing numbers to a hundred. An excited chorus shouted, “Yeah!” All students reached 100 successfully, and some went beyond.

In this episode we stress that the teacher used patterns of variation to draw attention to the necessary features required for students to count and write numbers to 50. She also used both visual and auditory cues to emphasize what was changing from example to example. As in the previous episode, some assessment was based on individual or choral response. In contrast to the previous episode, however, she also frequently assessed all students by asking them to indicate their responses on mini-whiteboards. The teacher responded to student assessment by offering necessary support and by moving faster than planned. By the end of the first segment of the lesson, the entire class was excited to continue writing numbers from 50 to 100 independently, with several going beyond.

Episode 3: Weak Variation, Medium Assessment, Low Response

This episode from a Grade 5 lesson focused on the number of sides and vertices in polygons; students experienced high success early in the lesson, but there were

no effective extensions that allowed students to build on that success. As a result, engagement remained low. This episode offers a marked contrast to the excitement in the previous episode on counting to one hundred. It appears that all students successfully counted the sides and vertices on various polygons that the teacher offered to open the lesson. However, they then spent considerable time counting sides and vertices on various shapes, without consideration, for instance, of whether this would *always* be the case—which may have triggered a more significant mathematical exploration. When a student asked if the number of sides and vertices would always match, the teacher replied, “Try it and find out”! but did not follow up to see how the student engaged with the question. The team member observing this class challenged some students to try to find a polygon that did not have equal numbers of sides and vertices, and many students spent considerable time exploring this possibility. However, it was not discussed in the larger group. Instead, the lesson moved on to defining “polygon” and to naming and sorting polygons with up to 5 sides. Again, students successfully completed independent work, but many finished very quickly and spent a good deal of time waiting as the teacher seemed to stick to the lesson plan instead of responding to student feedback that suggested that most, if not all, students were ready to move on to more challenging work.

Episode 4: Medium Variation, Strong Assessment, Strong Response

In this episode, we show how a Grade 1 teacher made numerous attempts to support students in representing numbers to 20 using unit and ten-blocks. The episode also shows examples of extensions for students who met the expected outcomes during class. In addition to providing several opportunities for each child to respond to questions, this teacher paid regular attention to one student, Heidi, who struggled far more than her classmates. Although the variation in the lesson provided in the teachers’ guide omitted a variation that emerged as significant, the teacher’s careful use of assessment, persistence in seeking an effective response, and effective adaptation of the given variation allowed everyone to reach understanding by the end of the lesson.

As in Episode 1, the teacher opened with a series of questions and took answers either in chorus or from individual students. It was clear to the teacher, however, that some students were giving wrong answers. In particular, there seemed to be confusion about the number of ten-blocks and one-blocks used to represent a particular number. The instructional sequence involved showing blocks for 11, 12, 13, 14, 15, and 18; students were to identify the number of ten-blocks and unit blocks, then write the number. Note that in this sequence, the number of units varied while the number of ten-blocks remained constant: There was always one ten-block.

After offering the first item in the sequence (1 ten and 2 ones), the teacher checked on each student individually. She then offered explanations for 10, 11, 12, and 13. At this point, she asked the class to represent 14 and checked each student’s response.

Heidi was struggling, so the teacher gave her 4 one blocks and one ten-block. Then she requested the attention of all students and provided an explanation on the board. The teacher asked students to show blocks for 18 by themselves. She walked through the class checking on students, offering assistance and posing further challenges (19 and 17). This gave her some time to work individually with some students. Only two or three students, including Heidi, still had trouble representing the numbers.

The class moved to the next part of the lesson, in which the students worked in their assessment and practice books. The teacher introduced this work by using a document camera to show an example solved in the students' book. She then asked individual students to solve the given exercises in front of the class. The page showed tables with 2 rows of 10, numbered from 1 to 20. In the initial example, the first row, with numbers 1 to 10, was marked with a darker color, as were the places corresponding to the numbers 11 to 18 in the second row (see Figure 1).

Figure 1. Example from the assessment and practice book

Underneath this image, the following sentence was written:

18 is 1 tens block and 8 ones blocks

The first practice exercise included a similar image with the top row coloured in and 5 blocks coloured in the second row. Beneath this was written:

15 is ____ tens block and ____ ones blocks

When the teacher asked, "How many ten-blocks," some students answered in chorus, "10." The teacher then explained that there was only one ten-block. She gave every student one ten-block, asking, "How many ten-blocks did I give you?" until every student answered one. Similar questions asked students to identify 17 and 11.

Subsequent questions required students to identify tens and ones without the support of individual charts: They were now supposed to place blocks on a given 20-chart to represent the given number, and then complete a sentence similar to the previous page. For instance, the first task was:

14 is ____ tens block and ____ ones blocks

One student showed the answer on the board to the whole class, and the teacher re-explained the use of one-blocks and ten-blocks. Subsequent tasks in the practice book asked students to consider 19, 11, 13, 12, and 20. Note that so far, all but one of the examples focus on a single ten paired with varying numbers of units (and that one example comes at the very end). Given that effective variation uses contrast to draw attention to critical features, it is perhaps not surprising that we observed students who struggled to name the number of tens blocks.

Students started to work individually and the teacher walked through the class checking on each student. At least three students, including Heidi, still had trouble differentiating the ten-block from the one-block. At this point, the teacher started giving these students more ten-blocks. She started with Heidi, showing three ten-blocks and counting, “One, two three.” For the first time, the pattern of variation now included contrasting numbers of ten-blocks.

The teacher went to another student, who had apparently written a 5 in the space for ten-blocks; she indicated that in the case of 15, there were not 5 ten-blocks, but only one ten-block. This student seemed to understand and corrected his work.

Another student had written 15 in the space for ten-blocks. The teacher showed her 15 ten-blocks in her hand while saying, “Here are 15 ten-blocks” and asking, “How many ten-blocks are there [in the given example]?” The teacher gave one ten block to the student, asking: “How many blocks did I give you?” The following dialogue ensued:

Student: One.

Teacher gave another: Now how many blocks?

Student: Two.

Teacher took away one block and asked: OK? How many ten blocks did I give you right now?

Student: One.

Teacher: Then write one.

The class finished, and the teacher asked Heidi to stay; she spent 4 minutes working with her.

The teacher held up ten ten-blocks: “This is ten ten-blocks.” She gave the blocks to her to hold in her hand.

Teacher: See ten blocks?

Heidi started giving the blocks back, one by one to the teacher. The teacher counted while the student was giving the blocks back: “One, two three tens.”

Teacher: Can you show me 5 ten-blocks?

Heidi: One, two three four five [moving the ten-blocks one by one]

Teacher: So that’s five ten-blocks. Now show me one ten block, just one.

[Heidi moved one block.]

Teacher: Now, how many ten-blocks do you have here?

Heidi: One.

Teacher: So, write the number one. How many one blocks? How many ones?

In this episode, we stress that the teacher led group discussions in which one student or students in chorus responded to her questions, instead of using an all-student response system. Nonetheless, it was clear to her that there was confusion about the distinction between one ten-block, and ten unit-blocks. The teacher prompted attention to this difference several times during the lesson, which included giving one ten-block to each student and asking, “How many blocks did I give you?” Still, some students were confused.

By the end of the lesson, the teacher started to vary the number of ten-blocks in her explanation to students. This seemed to be more effective for the students who were still confused. In the case of Heidi, this seemed to be more effective when she actually counted the blocks—as opposed to the first time when the teacher showed her the blocks. This use of contrast is a keystone of the variation theory of learning, which takes as its fundamental conjecture that new meanings must be experienced as difference against a background of sameness rather than as sameness against a background of differences (Pang & Marton, 2013).

DISCUSSION

The episodes presented here serve to illustrate the three teaching practices we wish to emphasize, corresponding to the three categories of the observation protocol: We acknowledge the existence of other aspects of teaching quality not addressed in this discussion, which have been omitted in order to highlight these categories. The first two episodes presented in the previous section serve to contrast the all-student-response approach to assessment encouraged at the intervention. While in Episode 1 the teacher only assessed responses from individual students or in chorus, the teacher in Episode 2 used the whiteboards to assess all students at the same time. In both episodes, the teachers assessed during class, but in the first, the teacher assessed students one at a time after they were working independently rather than as a group to determine readiness for independent work.

Two different teachers’ responses to student feedback are contrasted in Episodes 2 and 3. In Episode 2, the teacher modified her plan by moving faster in response to students’ feedback and comments, while in Episode 3 the teacher did not move on, even though students seemed to be ready for the next part of the lesson. The difference in student engagement was that in Episode 2, students were excited about counting beyond 50 by themselves, whereas in Episode 3 students were bored and therefore marginalized from expanding their mathematical understanding.

In Episode 4, the teacher was responsive to student feedback. She provided additional questions to students who completed the work early and attempted to address the confusion between one ten-block and ten one-blocks in different ways, including several explanations and giving one ten-block to each student. However, it was at the end of the lesson, when she tried to vary the number of ten-blocks that she seemed to find a more effective way of prompting attention to these differences.

Introducing this variation early during class may have prevented the need for individual remediation, thereby allowing *all* students to move faster, or to engage in more challenges. In future iterations of this lesson, this teacher may more quickly invoke the more powerful patterns of variation she uncovered through her work with students, allowing greater opportunities for early mastery and further extension for more students. Nonetheless, we categorized it as a strong example in which careful assessment and response coupled with appropriate adjustments to variation led to favorable outcomes for all students.

CONCLUSIONS

The framework developed for classroom observation described in this chapter served to identify teaching practices that can either support students' learning if used effectively or marginalize them from learning if not. The categories are interrelated: Continuous assessment of all students can lead to timely and effective responses to student feedback, and these may be informed by effective variation of critical features. It is important to notice that effective variation requires specialized mathematical knowledge to identify the critical features in each lesson.

We did not intend to critique teachers, but to identify teaching practices that prevent marginalization in terms of engagement in further mathematical learning. In this study, we have witnessed teachers with a strong commitment to their students, as evident in the way they responded to student assessment. However, for various reasons, the enacted lessons did not always effectively support mathematical learning for all students. We observed that the combination of assessment based on all-student responses and variation that prompts attention to critical features during instruction seemed to better support students' understanding in class. Additionally, teacher responses to student feedback can support both students who still struggle with the content of the lesson and those students who meet the expected understandings at a given moment.

Professional development sessions for participating teachers were included as part of the intervention, and some of the aspects discussed in this chapter were addressed in these sessions. Yet, we continued to observe marginalizing practices such as lack of continuous assessment, lack of teacher response to student feedback, and ineffective use of variation. In some cases, teachers who demonstrated an exemplary lesson for certain lessons received lower scores on other lessons. In continuing our research, we hope to better understand the mechanisms that could support teachers in enacting these practices in their classrooms.

While we have identified the three teaching practices described in this chapter, we are still in the process of refining and testing the complete descriptors for the levels in the observations protocol. Once developed, this protocol could serve to support teachers adopt these practices in the classroom through feedback from peers and external observers.

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