Special Issue on Numerical Tables and Tabular Layouts in Chinese Scholarly Documents

Part II: Synchronic and Diachronic Approaches to the Texts of Tables

Introduction from the Guest Editor

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The two issues of EASTM that we have devoted to tables address the same set of basic questions.¹ They examine tables in a historical fashion, focusing

¹ This issue (the second on the topic of numerical tables and tabular layouts published in this journal) also contains some of the papers presented during a two-day workshop I organized on March 22-23, 2012, on the topic of "Tables in scholarly Chinese sources" (<http://www.sphere.univ-paris-diderot.fr/spip.php?article626>). The workshop was held in the context of the project "History of science, History of text" and the ANR Research Project "History of numerical tables" (HTN), which are

in particular on the topics for which actors used tables, the reasons why they opted for this form of expression, and the variety of written forms that in different contexts actors created, or adopted, to compose these tables. The first issue centered on uses of tabular layouts in early imperial China in writings devoted to astral phenomena and past political events. The articles it contains brought to light two general aspects worthy of note. First, the production of a tabular layout requires (and also makes possible) specific operations in order to yield the content of the table and display its information. This led us to pay special attention to the operations that actors had to carry out to design and use layouts of this kind. Secondly, it appeared that tabular layouts were not merely used as textual forms convenient to arrange data and facilitate their retrieval. The first issue illustrated how actors used such layouts to explore questions and express specific meanings. These remarks suggest motivations as to why in some contexts this type of textual form might have seemed appropriate. Further, a conclusion emerged from the examination of how tabular layouts had been used. Through the use of rows and columns, layouts of this kind shaped specific parts of texts (one could even say, correlated parts of texts), which were read as such and also in relation to each other. In other words, this specific spatial inscription of the text allowed users to read different things in texts, and to read them differently. It further allowed users to rely on these lines to work with the tables.

This second themed issue concentrates more specifically on *numerical* tables, and focuses on questions raised by the synchronic and diachronic variability of the texts of tables for numerical tables that from our perspective as observers represent the same relation. Indeed, far from being “universally the same,” the textual forms that have been used to write down numerical tables have changed from context to context. Scientific texts provide abundant evidence of this fact. With these changes in textual forms, the operations carried out to make these tables and use them have changed accordingly. How can historians deal with this variability, and also, what

both developed within the research group SPHERE (Univ. Paris Diderot, Sorbonne Paris Cité, SPHERE, UMR 7219 CNRS, Univ. Panthéon Sorbonne, F-75205 Paris, France). I am grateful to the members of these projects as well as to the audience, in particular to the commentators on the articles, for their remarks. It is my pleasure to thank Dominique Tournès, the Principal Investigator of the HTN project, for his unflinching support over the years, notably in the organization of this workshop. The anonymous readers provided each author with many suggestions for improving his or her article. We collectively express our gratitude for this work. Heartfelt thanks to the editorial team of EASTM, and in particular to Hans Ulrich Vogel, Ailika Schin-köthe, and John Moffett, but also to Christine Moll-Murata and Ulrich Theobald.

does this variability mean for them? These are the main questions we consider. We will see that, from this perspective, we draw conclusions that bring us back to those reached in the first issue.

To begin with, with Li Liang's contribution, we consider these questions with respect to numerical tables written using tabular layouts (later, we turn to other types of textual forms used for numerical tables). Expressed in general terms, what Li shows is how tables and their layouts depend on the scholarly cultures in which they were produced, and also on the scholarly cultures of the intended users. Seen from a different perspective, this implies that the specific features of tables bespeak the context of their production and of the use for which they are designed. Li further argues that historians could rely on this dependence of tables and layouts vis-à-vis these scholarly cultures to address several general issues of importance for the history of science. In his case, the main issues at stake are those of the circulation of knowledge between different contexts, and also of the operations actors carry out to accommodate the newly acquired knowledge to their own uses in relation to their aims and their cultures. Numerical tables and their different tabular layouts prove to be a most interesting site for the analysis of these general phenomena.

Li focuses on numerical tables found in writings about astral sciences, and more specifically writings dealing with Islamic astronomy in China. Let us provide some information on the documents available, so as to emphasize the generality of the theses Li puts forward in this case study. Evidence allows historians to perceive the presence of Islamic astronomy in China from the Yuan dynasty (1271-1368) onwards. However, so far the available documents do not enable us to examine the *extent* to which Islamic astronomy was introduced in China, and the diffusion it may have had at the time. With the Ming dynasty (1368-1644), the documentary evidence becomes more important. It allows us to grasp the fact that, due perhaps to a stronger interest in astrology, and probably under the influence of the founding emperor of the dynasty Zhu Yuanzhang (1328-1398), technical knowledge that astronomers working in the Bureau of Islamic Astronomy specifically possessed began to be perceived as possibly useful for astrological purposes. In fact, in China the latitudinal deviations of celestial bodies from the ecliptic had not been an object of specific interest. Yet these deviations are useful to approach the question of the encroachment of bodies, a phenomenon essential to astrology in China. In the 1380s, Chinese astronomers like Yuan Tong 元統 (fl. 1380s) were sent to the Bureau of Islamic Astronomy to learn this body of knowledge and teach it to experts in the Chinese Bureau of Astronomy, and also perhaps to translate it. One can observe this process by relying on Bei Lin's 貝琳 (d. 1490) later edition in 1477 of Yuan Tong's *Huihui lifa* 回回曆法 (Islamic Astronomical System), known through a 1569 reprint. This first

piece of evidence of a translation of knowledge learnt in the Islamic Astronomical Bureau can be compared to another major achievement by Liu Xin 劉信 (d. 1449), the *Xiyu lifa tongjing* 西域曆法通徑 (Comprehensive Guide to the Astronomy of the Western Regions) compiled in the second third of the fifteenth century. This work seems to reflect the project of making Islamic astronomical knowledge available to Chinese practitioners, and in particular of helping them make astrological computations more easily. Two other sets of documents enable us to understand the presence of Islamic astronomy in China: Yuan Huang's 袁黃 (1533-1606) book, *Lifa xinshu* 曆法新書 (New Book of Mathematical Astronomy), written in the 1570s, and the various manuscripts and publications attesting to Mei Wending's 梅文鼎 (1633-1721) compilation of the "Monograph on the Pitch-pipes and the Calendar" for the Mingshi 明史 (History of the Ming).

The key point for us here is that the tables in all these documents basically have the same values and the same content. However, their layouts, and more generally their written forms, differ. Note that Li can interpret these differences as shedding light on features of the process of the reception of Islamic astronomy in China.

The entries of Yuan Tong's tables, as they can be perceived in Bei Lin's reworking of the book, were numbers in *du* 度 and *fen* 分. These were traditional units of measurement in China. However, in the process of translation, they were given a new meaning related to the sexagesimal number system that astronomers used in Islam. Note again that here variability of the text of the table does not lie in the expression, but in the meaning users should attribute to it.

These tables supposed the use of interpolation, the values thanks to which interpolation could be carried out being also provided in the table. In Liu Xin's work, by contrast, the massive expansion of the same tables (24 volumes, and no computation needed to use them, given the fineness of the steps between consecutive entries) can be interpreted as reflecting the fact that the Bureau of Astronomy began using these tables significantly for astrological purposes. Thanks to the latter tables, users could avoid computations and simply pick out data from the tables. Accordingly, despite the fact that the two types of tables lead to exactly the same results, variability in some of their features reflects two types of uses, and probably also two types of users, who had different competences, and whose operations to use the table were different.

The transformation of entries from the sexagesimal number system to a centesimal one in Yuan Huang's book, Li suggests, again appears to reflect his intended readership, that is, in this case a wider Chinese readership outside the specialized bureaus. Moreover, Li interprets the new shape of the tables in this writing as a possible reflection of Yuan's project to contribute gathering momentum to solve the problem of updating the *Datong*

calendar – the Chinese calendar that had become hopelessly outdated, but that the Bureau of Astronomy did not manage to transform satisfactorily. This task required looking for sources of knowledge everywhere, including in Islamic astronomy, and attempting to put it in a format in which Chinese readers could conveniently use it. In any event, the format of Yuan's tables reflects the circulation of Islamic astronomy outside the official bureaus. Incidentally, maybe the transformation of the format of the entries also indirectly indicates the problem posed to Chinese users by the fact that Islamic tables were based on a sexagesimal number system. In this case, the change of number system to write down the entries, at a time when a wider readership was intended, might likewise retrospectively shed light on the rationale behind Liu Xin's massive expansion of the Islamic tables. Perhaps, Liu Xin aimed to relieve the burden of computations on users who were not versed in computations with a sexagesimal number system. From this perspective, we can grasp part of the knowledge that the use of some tables requires.

Finally, Mei Wending's treatment of the Islamic calendar in the *Mingshi* is interesting since it illustrates a case of change in the layout of a table. The project of reporting about the essence of Islamic astronomy in the context of a historical project might have motivated his intention to compress the tables. This might be the rationale for his adoption of a format introduced by Jesuits in the Chinese books in which they explained the astronomy coming from the West. This format is characterized by the fact that it relies on the symmetry of data to divide the extension of a table by two. Knowledge about the structure of the data is brought into play to transform the layout of the original table. However, consequently, users of the transformed table had to learn a new way of handling tables to be able to use them. In this case, handling includes a new scholarly circulation in the text of the tables, typical of another scholarly culture.

Further, the adoption of a tabular format found in Western sources to produce a historical account also led Mei Wending to write numerical values horizontally, degrees being placed in a cell separated from minutes and from seconds. However, interestingly enough, the degrees were written to the left of the minutes, themselves to the left of the seconds, in a format unknown to Chinese users prior to the introduction of writings from the West.

This episode illustrates how resources from one scholarly culture can be adopted by another scholarly culture for wholly different purposes (in the former context, tables are used to compute, in the latter, they are used to present bodies of knowledge from the past). From a different perspective, the new intended use for the Islamic tables of the past can also be correlated with specific changes in the management of numerical values. Their inclusion into a historical book did not aim at providing tables as tools for computation. Accordingly, the values formerly inserted in the tables to help carry out interpolations were deleted.

All these changes show the malleability of the texts of tables. This explains why these texts reflect the contexts of production as well as the projects for the tables and the intended user. In this process, features of the texts of tables circulate across scholarly contexts and get recycled.

Another feature that Li emphasizes likewise casts an interesting light on the circulation of characteristics of tabular layouts across contexts. Owing perhaps to old traditions of representation of numbers in China, some Chinese (and also Arabic) sources indicated values to be subtracted or added using red and black ink, respectively. In the context of printed Chinese tables for astronomical purposes, the opposition between these two types of value took the shape of writing characters either in black in a white cell, or in white in a black cell. In later tables, this device was replaced by a dividing line, which separated the two sets of numbers. It may be that the device of the dividing line was used to distinguish northern values from southern values in Islamic tables. This feature, specific to Islamic astronomy, can be found in tables in Chinese that are reporting about Islamic astronomy, and thus use the same artifact of the dividing line. However, the artifact is picked up for other types of tables, to delineate between values to be added and subtracted. So, in this new context, the artifact is re-used, but with another meaning and for other purposes.

In conclusion, the circulation of diagrammatic features of tabular layouts from one context to the other, and other similar examples evoked in the previous issue, all illustrate a general phenomenon, meaningful for the history of science. Actors introduce cultural practices in the context of specific activities, and to solve specific problems. Once these practices have been introduced, they can be picked up and used for other purposes, either in the same scholarly context, or in others. The creation of these devices and their circulation further show two important points. First, they illustrate how actors shape the scholarly cultures in the context of which they operate. Second, what circulates between contexts is not only concepts or results, but also specific cultural practices and ways of working.

So far, the variability that we have considered in the texts of numerical tables derived from the fact that their tabular layout differed and that they required different types of circulation. It also derived from the fact that their entries could use different number systems and different modes of inscribing numerical values, and that they conformed to progressions using coarser or finer steps. In his article, Li emphasizes yet another kind of difference between texts of tables that is of more general interest for us. He describes the shift from one-dimensional formats for tables (lists with specific layouts), typically used in some early Chinese astronomical tables, to the two-dimensional (tabular) formats, usually employed in Arabic tables. This remark points out another type of variability for the texts of numerical tables, which is crucial for the second article of this issue. Indeed, although numerical

tables can be textualized in a tabular format, this is not the only type of textual form for writing them down to which our sources attest.

This need not surprise us: this point simply derives from the double meaning that the English word “table” can take. The term can refer to a “tabular layout.” This is the meaning with which I have so far used the term. However, “table” can also refer to a specific type of text, as in the expression “multiplication table.” A “multiplication table” represents one way of expressing the relation that the operation of multiplication establishes between a pair of numbers and their product. Multiplication tables do so in the form of a list of similar clauses, where each clause states the result of a given multiplication in a sequence of multiplications. This second meaning of the term “table” is important for our topic. Indeed, as has just been mentioned, in the corpus of astral sciences in Chinese, numerical tables – with the latter sense of “table” – can be textualized with a tabular layout (the two meanings of “table” being then used in relation to a single text) as well as with a list of clauses. The same fact also holds true for numerical tables in mathematical documents in Chinese. Moreover, some of the documents that Ma Biao’s article mentioned in the first issue were in fact also tables in this sense.

The key fact is that, depending on the Chinese documents considered, what for us appears to be “the same” numerical table has been textualized in one way or the other. What can account for the choice by various actors of either one of these types of texts? This is one of the questions I address in the article that I contribute to this issue, and that is devoted to numerical tables in Chinese mathematical documents composed in China between the early decades of the Empire to the Song-Yuan dynasties. Clearly, examining this question can help us understand better what, more generally, might have motivated actors when they opted for the use of a tabular layout.

The earliest known Chinese writings devoted to mathematics are manuscripts written on bamboo slips or on wooden boards in the first decades of the Chinese empire. They were excavated from tombs or pits, or else bought on the antiquities market. These manuscripts present several properties regarding numerical tables that are worth pondering. By contrast with later texts handed down through the written tradition, all the manuscripts of this period the text of which is available to this day (2016) abound in numerical tables. However, none of these tables was written using a tabular layout – and more generally, they employ no tabular layout. My contribution establishes that there existed two specific types of textual format that scribes used to write down numerical tables. One made use of registers, while the other made specific use of a punctuation mark (hook sign). The key point is that, if we set aside an exception, each manuscript seems to have employed a single way of writing down tables.

Establishing the textual formats used to inscribe tables enables us to see that each mathematical manuscript was composed of only two types of components: text written in a continuous fashion (usually problems and procedures) and numerical tables. Moreover, this allows us to determine which topics were presented in the form of numerical tables. These include multiplication tables for measurement units, for powers of ten, for digits or for fractions. These also include tables for conversion and numerous tables related to the management of grain. Among the latter tables, we find tables presenting data and tables gathering numerical procedures, which probably reflect actors' operations with grains. What, more generally, does the presence—or the absence—of numerical tables tell us about the mathematical culture and the social context in which these writings—or later writings—were produced? What does the variation in the texts of tables tell us about how the tables and their texts were used? These are some of the specific questions that my contribution raises.

But the main question lies elsewhere. The previous assertions seem to be contradicted by the recent discovery (unfortunately on the antiquities market) of another document from roughly the same time period, in which a multiplication table is inscribed on twenty-one slips, using a tabular layout.

Why did, in this case, actors opt for this type of layout? My suggestion for answering the question relies on several observations. This numerical table was not inserted in the context of a larger writing, but seems to have been used separately. Furthermore, silks threads appear to have been attached to holes made in the slips. Most probably, actors wielded them relying on the horizontal and vertical lines of the tabular layout to pick out data. I thus suggest that this document was an object, and more specifically a computing tool, rather than a numerical table included into a writing, as the other numerical tables clearly were. This explains why, in my view, this artifact does not contradict my conclusion that early mathematical writings do not evidence the use of tabular layouts to inscribe numerical tables.

Moreover, this artifact invites us to discuss further what was at stake in a tabular layout. The spatial structure of the data on these twenty-one slips shaped horizontal and vertical lines on which actors seem to have relied to carry out operations. This can be compared with the diagrammatic features of the tabular layouts, the use of which was described in the articles of the first themed issue. In the case of the multiplication table, rows and columns were not meant to be *read*. Neither did rows and columns shape textual parts with the intention that they should be interpreted. Rather, lines and columns were created to enable the user's actions. Another remark suggests the same conclusion. On the one hand, this numerical table differs from other similar tables because it does not make use of the symmetry of multiplication to decrease the number of clauses included in the text (as we have seen Mei Wending also did with a tabular layout). On the other hand, it does compress

the text of the table, since for all the clauses in a column, the multiplier is the same, and it appears once, on top of the column, whereas for all the clauses in a row, the multiplicand is the same, and it occurs only once, on the right slip. Compression appears as a value in different contexts, but it can be achieved using different means. Note that in the case examined, this way of arranging data might have placed on top of each other various partial results that had to be added to yield the final result. If this were the case, it would show another way in which the spatial layout was correlated with the user's expected operations. In conclusion, the diagrammatic characteristics of this specific tabular layout seem to have been designed in relation to a use of the table for action. More generally, we see that layouts of this type have been used *both* as modes of expression and tools of action.

My study of Dunhuang mathematical manuscripts shows a situation for the textualization of numerical tables virtually identical to that which I have described for earlier manuscripts, except for a change in the material support for writing. This points, I think, to a form of continuity that has not been mentioned so far. As for the earliest known printed mathematical writings, the numerical tables they include often use textual forms similar to those evidenced in manuscripts. However, probably around the eleventh century, a shift appears to occur. Although mathematical books still contain numerical tables similar to those described for manuscripts, we also see appearing *within* the books numerical tables with text that makes use of tabular layouts. A closer examination of these tables, and of their relationships with their environment in the writing within which they occur, shows that the mathematical use of these tables relies on the lines (horizontal, vertical or oblique, which are sometimes drawn) that their tabular layout displays. In this case, as in the case of the computing tool, the diagrammatic features of the tabular inscription are essential to the operations that practitioners will carry out on the text of the table. What is new is that these diagrammatic features are now also used to shape subsets of clauses and state mathematical meanings on these subsets and on the relations between them.

In addition, it is striking that a similar phenomenon takes place for visual tools. Everything seems to indicate that the visual tools to which the earliest mentions in mathematical writings refer were material objects, like the computing tool described above. Similarly, the use of these visual tools included operations (in this case cutting, moving around, rearranging, and so on). However, starting *also* from *ca.* the eleventh century, visual tools appear to have been inserted into writings in the form of illustrations, and referred to as *tu* 圖. What is even more striking is that the numerical tables inserted into mathematical writings, for which a tabular layout is used, are also designated as *tu* 圖.

These remarks suggest that actors might have perceived a similarity between tabular layouts and visual tools. This similarity is manifest through

the terms used as well as through the history of the material realization of these two types of artifacts. In fact, a third similarity can be added to these: it relates to the positions in which both diagrams and tabular layouts are placed in mathematical texts. This issue raises questions that need to be further explored. However, this already suggests a tentative conclusion: when actors used tabular layouts to write down tables, the diagrammatic features of these types of text were essential to how actors perceived them and dealt with them.