Karine Chemla, Renaud Chorlay and David Rabouin (eds.)


This collection of essays deals with the problem of generality in mathematics and the natural sciences – an issue that is of the greatest importance. The nature, aims, limitations, and possibilities of generality have been a topic of discussion among scientists, philosophers, epistemologists and logicians for centuries. The present volume offers a fresh perspective on the matter. The general aim of this collection of papers is clearly stated by the editors in the introductory chapter, which in itself constitutes an important essay in historical epistemology. Instead of trying to provide a unified theory of generality in the sciences, or offering the reader an array of different, competing contemporary perspectives on the problem, the volume chooses a historical path for its approach to notions of generality. The main point of the book lies in the presentation of the numerous and diverse conceptions of generality that have been accepted and used from the beginnings of ancient science up to the most recent research. The volume also sets out to show how quite different notions have been held, within a variety of disciplines, about what should be considered a ‘general’ result, theorem, or law. Overall, the aim of the book has been fully achieved.

The authors provide us with an impressive collection of historical theories and practices of generality, ranging from ancient Greece to the works of Banach, Poincaré or Ranvier, and from pure mathematics to physics, the theory of dynamical systems, zoology, and anatomy. By so doing, the authors attempt to avoid any trivialization of the different meanings of generality by subsuming all of them under the same (vague or imprecise) notion, and to investigate their differences. At the same time, the contributions to this book also show how these differences between conceptions of generality were affected by the historical contexts and the common epistemological views of a given age, the specific ideas of any given scientist, as well as by the specific aims for which a particular generalization was performed and theorized. The presentation of this great variety of approaches is outstanding, and taken together, they offer a very rich, almost overwhelming, historical picture of the manifold ways in which scientists and philosophers have conceived of the notion of generality.

Such a variety may be investigated in a diachronic perspective, by showing how the notion of generality changed over time in the course of the development of a single discipline, and how the interplay between continuity and rupture shaped the further understanding of what should count as ‘general.’ Let us here only mention the theory of functions from Lagrange to Cauchy as one
example given in this book. However, a synchronic approach to the same topic is also possible, which may show how different scientific traditions in different countries have conceived of generality, in the same period and with reference to the very same problems, albeit in radically different ways (take Poincaré and Maxwell on electromagnetism). This second, synchronic approach may also show how disciplines that are closely related to one another yet often conceive of generality in radically different ways, while sciences that otherwise bear little relation to one another may, equally surprisingly, share similar notions of what constitutes generality. The book thus places great emphasis on episodes of circulation and contamination of different notions of generality, and on re-interpretations of the meaning of generality (often by mistake, and often with excellent results). The volume also looks at debates and controversies that arose between adherents of different epistemologies or between practitioners of sciences that tended to look at each other with skepticism or suspicion (say, biologists’ skepticism vis-à-vis model-making in mathematical physics). In doing so, the volume steers clear of attempting to describe a naively linear development of the notion of generality.

The book’s emphasis on the practices of generalization is of special importance. All contributions deal with actual scientific cases and the role of generalization in dealing with scientific issues; no reference is made to any abstract, disembodied epistemology. The authors are interested instead in the actors practicing generality. The few philosophers that are extensively discussed in the volume (Aristotle, Descartes, Leibniz) are considered, above all, insofar as they were scientists, and their ideas are situated in the contexts of the scientific debates to which they contributed. The main protagonists of the book are mathematicians and natural scientists, and specifically those who invented new representations of generality, critical of other representations, or who reflected on the epistemology of their own discipline. This is a very fruitful perspective, since it allows the book’s authors to explain the genesis of each new notion of generality that they discuss by reference to concrete scientific examples and problems.

Given the specific approach that is favored by the editors of the volume, the reader should not expect to find a complete treatment of the subject. As will have become evident by now, this volume offers a selection of topics and case studies in the history of the notion of generality instead of attempting to provide a general outline or a broad sketch of important historical developments. As a consequence, a number of topics that are debated in the current literature on the history of epistemology are not considered in this book. As has already been mentioned, philosophers and logicians receive little attention, even where their work has played an indisputable role in the history of the notion of
generality. We find no traces of Locke, Berkeley, Mill, Frege, and other authors of this caliber. This book even omits several extremely important scientific theories that are directly connected to the notion of generality, such as Klein’s Erlangen Program; or the nineteenth-century generalization of the notion of energy.

However, I do not think that these omissions are overly damaging to the book’s purpose. Apart from the fact that one should never criticize a book for its omissions, but instead appreciate its contributions – which, in this case, are a lot – the lacunae can be easily explained by the aim of the volume, which is – as mentioned – not completeness, but rather that of providing a map that guides the reader through still largely unknown territory.

The editors have chosen to focus on the domain of pure mathematics, where the development of the notion of generality can be studied in a particularly neat way, and with remarkable consequences for the epistemology of that science (ten papers out of 18). Several other essays are dedicated to physics (three papers) and the life sciences (four papers). By contrast, the problem of generality for historiography and for the human sciences is not dealt with, but the editors provide a few bibliographical indications in this direction, referring mainly to M. Hagner and M.D. Laublicher, eds., Der Hochsitz des Wissens: Das Allgemeine als wissenschaftlicher Wert (Zurich and Berlin, 2006). Several contributions deal with the notion of generality in the modern era (nineteenth and twentieth centuries), while a few others touch on previous centuries, taking us as far back as ancient Greece. The editors also offer a bibliography providing general information on research into generality as related to other cultures and times; one of the editors, Karine Chemla, has, in fact, written extensively about generality in ancient Chinese mathematics. Special attention is accorded to the views of Leibniz and Poincaré, whose ideas are discussed in two essays each; they appear to have been especially relevant in the development of new conceptions of generality.

After the editors’ important epistemological and methodological introduction (Chapter 1), the book continues with a remarkable essay by Karine Chemla on Michel Chasles (Chapter 2), who not only explicitly discussed various notions of generality, but also carried out a historical reconstruction of the different ways of representing generality in geometry, from the ancient Greeks right up to his own days. The essay on Chasles represents a perfect introduction to the volume’s topic, and documents that a historical approach to the notion of generality was already possible in the nineteenth century. Chasles, for one, used this approach to shed light on what he was attempting to do by generalizing certain theorems in projective geometry. Despite certain misrepresentations, Chasles’ historicizing analysis contributed to his own views on
generality. Remarkably enough, he did not view the development of the notion of generality from antiquity up to his own time as a linear route, but as an enterprise that was constantly interrupted and, which followed roundabout paths. Among the most interesting passages in the book are those that discuss Chasles’s diagnosis of the superior generality of algebra with respect to geometry, which discuss the possibility of arriving at a different type of generality in geometry through the introduction of the notion of a projective transformation, and which provide subtle analysis of the slow emergence of a geometry of space and of positions (that is, projective geometry) out of the geometry of magnitudes that was predominant in ancient Greece. These passages lucidly depict an actor of science in the act of practicing, simultaneously, mathematics, epistemology, and historical research.

The book continues with an essay by Eberhard Knobloch, on another important figure, namely Leibniz and his notion of generality (Chapter 3). Knobloch points to the connection between Leibniz’s conception of generality and his notions of order, symmetry, and harmony. This particular nexus of ideas shows how distant the concept of generality was for Leibniz, who was also a great logician, from a mere abstraction or the indeterminacy of a genus, and how as a mathematician he connected the notion of generality to that of a richness of structures and the fecundity of a theory.

In Chapter 4 David Rabouin discusses the notion of generality in Euclid from the specific perspective of a mathesis universalis, i.e. the problem of whether there exists a mathematical science that encompasses both arithmetic and geometry. Rather than asking whether, say, a Euclidean theorem proven on a certain triangle may be extended to other triangles (which is an epistemological question that is commonly discussed in the literature), Rabouin asks whether specific mathematical techniques – such as the Eudoxian theory of proportions – may be safely applied to both discrete and continuous quantities (that is, numbers and geometrical magnitudes), and how such a mathematical practice was created, developed, and theorized in the fourth century BC. This chapter thereby avoids the pitfall of trivializing the question of generality by reducing it to the generalization of an abstraction. Instead, it is fruitfully placed within the concrete context of the scientific practice of a specific era.

Chapter 5, by Igor Ly, which concludes the first part of the book, offers a reading of Poincaré’s thoughts on generality. The book’s conceptual line is faithfully followed in this chapter, which shows how Poincaré carefully distinguished between the merely ‘logical’ generality of an abstraction and the concrete generality of mathematics, which is quite different from the mathematical ‘generalities’ discussed previously in the book, with respect to Euclid, Leibniz and Chasles: for Poincaré it is based on induction and interpolation, and is
connected to probability in a rather complex way. In this essay Ly also discusses Goodman's and Wittgenstein's famous paradox of induction in its relation to Poincaré, attempting to show a mathematical way out of the problem.

The rest of the volume deals with more particular and specialized topics, of which no full account can be given here. The discussion of generality in mathematics continues with essays by Frédéric Jaëck on Banach's work in functional analysis (Chapter 7), by Emily Grosholz on Leibnizian analysis (Chapter 11), by Renaud Chorlay on nineteenth-century analysis (Chapter 14), by Evelyne Barbin on Descartes and Fermat (Chapter 15), by Frédéric Brechenmacher on Jordan and Kronecker (Chapter 16), and by Jacqueline Boniface on Kummer (Chapter 18). Physics and mathematical physics in turn are tackled by Anne Robadey, who writes on Poincaré and the recurrence theorem (Chapter 6), by Tatiana Roque on dynamical systems (Chapter 10), and by Olivier Darrigol on Maxwell (Chapter 12). A number of problems of generality in the life sciences are discussed by Yves Cambefort, who addresses the notion of genus in zoology (Chapter 8), by Stéphane Schmitt, who addresses the notion of homology in biology (Chapter 9), by Jean-Gaël Barbara, who investigates anatomy from Bichat to Renvier (Chapter 13), and by Evelyn Fox Keller, who scrutinizes the role of models in physics and biology (Chapter 17).

The contributions do display some qualitative differences, but their general level is very high. Most of them may, in fact, be read in their own right, as self-contained essays in the history of science. The accessibility of the essays is such that not only the specialist, but anyone engaged in the history of science or one of the book's historical key figures will benefit from them. However, the technical complexity of several of the essays and the scattered nature of the volume as a whole mean that the book cannot be recommended as an introduction to the problem of generality. Its ideal audience will be composed of historians of science with epistemological interests, and epistemologists wishing to engage with historical matters.

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