Mathematics, Mental Imagery, and Ontology: A New Interpretation of the Divided Line

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Abstract

This paper presents a new interpretation of the objects of dianoia in Plato's divided line, contending that they are mental images of the Forms hypothesized by the dianoetic reasoner. The paper is divided into two parts. A survey of the contemporary debate over the identity of the objects of dianoia yields three criteria a successful interpretation should meet. Then, it is argued that the mental images interpretation, in addition to proving consistent with key passages in the middle books of the Republic, better meets those criteria than do any of the three main positions.

Keywords

Plato – Republic – dianoia – hypothesis – mathematics

In Book 6 of Plato's Republic, Socrates uses the image of the Divided Line to distinguish four mental conditions, two associated with the visible realm and two with the intelligible. He differentiates the subsections of the visible segment of the line, eikasia and pistor, by the objects grasped by the soul in each. However, when he marks the distinction between dianoia and noësis, the conditions associated with the intelligible segment, he contrasts them based upon their methods of inquiry. Socrates tells us that dialectic, the epistemological method of noësis, directly grasps the Forms, but he makes no mention of a unique class

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of objects associated with dianoetic reasoning, a methodology he associates with mathematics. Consequently, there has been a history of controversy within Plato scholarship over the identity of the objects of dianoia, particularly the objects of mathematics, with disagreement as to whether Plato intended them to be intermediates as described by Aristotle, Forms, or sensible things used as images of the Forms.

This debate has brought to light serious problems with accepting any of the major positions. After surveying the main arguments of the contemporary debate and presenting compelling arguments against each, I shall propose a new alternative. Agreeing with proponents of the intermediates that the mathematicians grasp intelligible entities that are not Forms, I shall argue, along with supporters of the sensible things interpretation, that the objects of dianoia share the same ontological level as the objects of pistis, thus avoiding adding an ontological level Plato never explicitly endorsed.

1 Contemporary Debate Concerning Objects of Dianoia

There is a long tradition of interpreting the objects of dianoia as ontological intermediates.1 This position relies heavily upon Aristotle’s testimony, for in Metaphysics 987b14-18 and 1028b18-21 he directly attributes a theory of intermediates (τὰ μεταξὺ) to Plato.2 According to Aristotle, Plato believed that the mathematicals (τὰ μαθηματικά) existed at an ontological level between the Forms and the sensible things. Describing Plato’s view, Aristotle writes:

Further, apart from both the perceptibles and the Forms are the objects of mathematics, he says, which are intermediate between them, differing from the perceptible ones in being eternal and immovable, and from the Forms in that there are many similar ones, whereas the Form itself in each case is one only. (987b14-18)3

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1 This tradition may be traced back to Proclus, In Eucl. See particularly 4.14-5.10 and 10.16-11.25. More recent scholars holding this view include J. Adam (1902) 159-163; H.S. Arsen (2012); J.A. Brentlinger (1963); M. Burnyeat (1987) and (2000); 33-37; M.J. Cresswell (2012) 302-305; W.F.R. Hardie (1936) 49-65; M. Miller (2007) 319; D. Sedley (2007); and A. Wedberg (1955).

2 For other mention of intermediates, see 987b29, 991a4, 991b29, 992b16, 993b15-18, 997a35-b3, 998a7, 1002b13, 1059b3-9, 1069a34-6, 1076a17-21, 1077a11, 1080b11-14, 1080b23-25 and 1090b32-36.

3 Translation from Reeve 2016.
So, intermediates would be eternal, unchanging intelligible objects like the Forms, differing only in the aspect that they, unlike the Forms, are not unique.

One of the main arguments for intermediates is that Plato’s metaphysics requires them due to the Uniqueness Problem. Recognition of the Uniqueness Problem can be traced back to Cook Wilson’s 1904 argument that the doctrine Aristotle referred to as “ἀσύμβλητοι ἀριθμοί” arose out of the fact that Forms are unique and thus cannot be the objects of mathematical operation and knowledge. For example, there is only one Form of Two, or, one twoness. In arithmetic, we make statements such as 2+2=4. However, one cannot add twoness to twoness, because twoness is unique. This is why Aristotle describes the ideal numbers as “ἀσύμβλητοι,” translated as “incomparable” or “inaddible.” We encounter the same problem with figures. Forms of geometrical figures, too, are unique, so, for instance, circularity cannot intersect circularity. Reasoning along these lines, many contemporary figures have argued as follows. Plato believed the theorems of the mathematical sciences are true. Due to his metaphysical commitments, these theorems, which must be true of something, can be true neither of Forms nor of sensible objects. Therefore, Plato’s metaphysics demands the existence of intermediates.

The Uniqueness Problem provides only limited support for the claim that Plato held a theory of intermediates. As J.A. Brentlinger points out, this argument is indirect, for it “is evidence for what Plato should have thought, or could have consistently thought, rather than for what he actually did think.” And, Plato does not himself use this reasoning in the dialogues. Other popular arguments given in support of Plato’s intermediates are based upon the cave

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4 Annas (1975, p. 151) terms the problem formulated by C. Wilson “the Uniqueness Problem” in “On the Intermediates”.
5 Note that though Wilson’s reasoning is used to support the intermediates interpretation, he himself did not believe that Plato intended the objects of dianoia in the Divided Line to be intermediates. See C. Wilson (1904) 257-258.
6 C. Wilson (1904) 249-250.
7 C. Wilson (1904) 250.
8 Versions of this argument are found in H.S. Arsen (2012); J.A. Brentlinger (1963) 159-161; M. Burnyeat (1987) 221-222; M.J. Cresswell (2012) 95-96; W.F.R. Hardie (1936) 50; A. Wedberg (1955) 51-56; and C. Wilson (1904) 249-251.
9 J.A. Brentlinger (1963) 159.
allegory, Socrates’ treatment of faculties in Book 5, and Plato’s broader usage of αὐτό within the Republic.

The tradition of interpreting the objects of dianoia as intermediates has been challenged by such figures as Reginald Hackforth, Francis Cornford, Richard Robinson, and Sir David Ross, who posit that they are Forms. Proponents of this view argue that the Divided Line is based on a contrast between the visible and intelligible realms, and when this contrast is first introduced in lines 507b9-10, the intelligibles are explicitly identified as Forms. So, they conclude, it is reasonable to expect that both parts of the intelligible section of the Line have Forms as their objects. In addition, advocates of this position draw on 510d7-e1 in support for their interpretation. Describing the actions of the contemporary geometers, Plato writes: “They make their claims for the sake of the square itself (τοῦ τετραγώνου αὐτοῦ ἕνεκα) and the diagonal itself (διαμέτρου αὐτῆς), not the diagonal they draw.” Since Plato typically uses the qualification αὐτό, or “itself,” to indicate reference to a Form, Socrates’ words are often taken as proof that Forms are the objects of dianoia.

Though this group of interpreters agrees that the objects of both dianoia and noësis are Forms, there is disagreement between individuals as to whether the Forms grasped by dianoia are different from those grasped by noësis. On one view, the objects of the intelligible subsections are distinguished purely by the procedures by which they are studied: whereas the objects of noësis are known in relation to the Good and the other Forms, the objects of dianoia are

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For versions of this argument, see J. A. Brentlinger (1963) 156 and M. Miller 319. For an interpretation of this passage which argues against ontological intermediates as the objects of dianoia, see J.T. Bedu-Addo (1979) 103-105; J.S. Morrison (1977) 228-229; P. Pritchard (1995) 101-103; and N.D. Smith (1996) 37-39.


M. Burnyeat (2000) 35-37 and N. Denyer (2007) 304-305 argue that Plato often uses “itself” to remove some qualification or “clutter” and could therefore be referring to intermediates rather than Forms in 510d.

Scholars holding this view include F.M. Cornford (1932) 37-52; R.C. Cross and A.D. Woozley (1964) 230, 238; R. Hack forth (1942) 2-3; N.R. Murphy (1951), 166 ff.; R. Robinson (1953) 197; D. Ross (1951) 46-47; and C. Wilson (1904) 257-258.

This argument is found in N.R. Murphy (1951) 167 note 2; R. Robinson (1953) 197; and D. Ross (1951) 60.


See R.C. Cross and A.D. Woozley (1964) 236; R. Hack forth (1942) 3; R. Robinson (1953) 197; and D. Ross (1951) 59-60. Also relevant are Socrates’ references to “αὐτό τὸ ἔν” at 524e6 and 525de.
grasped only in isolation.\textsuperscript{18} The alternative view, however, holds that different Forms are studied on the two levels.\textsuperscript{19}

Nicholas D. Smith is perhaps the best known champion of the third interpretation of the objects of \textit{dianoia}, the view that they are sensible things used as images of the Forms. Smith agrees with the intermediates interpretation that the objects of \textit{dianoia} are images of Forms. However, he disagrees with the claim that they are ontological intermediates. Smith points out seven places in the divided line passage in which mathematicians are distinguished from dialecticians by their use of visible things as images.\textsuperscript{20} For example, Socrates describes the soul “using as images the things that were imitated before” (510b4-5) and “using as images those very things of which images were made in the section below, and which by comparison to their images, were thought to be clear and valued as such” (511a6-8). He reasons that, since the objects of \textit{dianoia} are images of the Forms, and the images of Forms associated in the divided line passage with the level of \textit{dianoia} are sensible things, a charitable reading of Plato should conclude that the objects of \textit{dianoia} are sensible things used as images of the Forms.\textsuperscript{21}

Smith believes that Socrates’ use of αὐτό in 510d-e refers to Forms.\textsuperscript{22} However, he frames the discussion differently than did the Forms interpretation, considering various meanings of the question: “What are the objects of \textit{dianoia}?” On the one hand, we could be asking which objects the mathematicians think about. On the other hand, we could be asking which objects are most aptly associated with the epistemological approach used at this level of the line.\textsuperscript{23} Smith believes that, in the interest of preserving the image/original relationship demanded by the line, the relevant question to answer is the second. The answer to this question is sensible things used as images of the Forms, such as the visible diagrams used by geometers. The objects of \textit{dianoia} have the same extension as the objects of \textit{pistis}, but the two classes have different intensions.\textsuperscript{24} Whereas the soul in \textit{pistis} views these objects as visible things, the soul in \textit{dianoia} views them as images of the Forms. Other supporters of

\textsuperscript{18} R. Hackforth (1942) 2. Others sharing this view include R.C. Cross and A.D. Woozley (1964) 238; R.L. Nettleship (1961) 249-258; and J.C. Wilson (1904) 259.

\textsuperscript{19} See Cornford (1932) 38-9; Robinson (1953) 195-197; and Ross (1951) 63.

\textsuperscript{20} N.D. Smith (1996) 34. He cites the following passages: 510b4-5; 510b7-9; 510d5-6; 510d5-6; 510e1-511a1; 511a6-7; 511c1; and 511c7-8.

\textsuperscript{21} N.D. Smith (1996) 35-36.

\textsuperscript{22} See Bedu-Addo (1979) 101-102 for a similar view.


this general position include J.T. Bedu-Addo, Robert J. Fogelin, J.S. Morrison, and Paul Pritchard.  

Each of these three positions is subject to compelling objections. Though the intermediates interpretation provides the best account of how mathematicians perform mathematical operations, and though it is supported by Aristotle’s testimony, Plato’s silence on mathematical intermediates raises serious concern. Since incorporating intermediates into his metaphysical framework would have been a major innovation, it is likely that, had Plato had held a theory of intermediates at the time of writing the Republic, he would have explicitly introduced it. And, the divided line passage, concerned with both ontology and mathematics, would have been the logical place to do so. However, we find no explicit discussion of the doctrine in the Republic. Indeed, in the Republic Socrates does not use the one/many distinction to distinguish Forms from ontological intermediates that are fellow inhabitants of being but instead to distinguish them from their likenesses in becoming (475e ff., 507a-b, 596a). Many scholars take Plato’s silence as evidence that he did not hold a theory of intermediates in this dialogue. At the very least, Plato’s silence should give us pause.

The interpretation that the objects of dianoia are Forms has been offered as an alternative, but it suffers from two serious problems. Though this interpretation appears to have strong textual support in 510d, it is inconsistent with the ratio of the divided line. Based upon Socrates’ instructions for drawing the line, its proportions require the objects of dianoia to image those of noësis just as the objects of eikasia image those of pistis. However, Forms cannot image...
Forms. Second, if we accept the Forms as the objects of mathematics, we are faced with the Uniqueness Problem. So, the Forms would appear to be excluded as candidates due also to the fact that they are not the type of entities on which one may perform mathematical operations.

Though the sensible things as images of the Forms interpretation can account for images of the Forms at the level of dianoia, it too faces the Uniqueness Problem. According to this interpretation, sensible things used as images of the Forms are objects of dianoia in the sense that they are the “things with which the thinker at that level are most aptly associated, in virtue of their epistemological approach.” Forms, however, are the objects that the mathematicians think about. The sensible things as images of the Forms interpretation is no better equipped than its predecessor to solve the Uniqueness Problem.

In summary, although each of the three main interpretations has strengths, each fails in its attempt to provide an account of Plato’s objects of dianoia in the divided line. The intermediates interpretation must appeal to ontological entities which Plato does not explicitly mention, the Forms interpretation fails to account for how the objects of dianoia function as images of the Forms, and both the Forms and the sensible things as images of the Forms interpretations fail to account for the repeatability of mathematical objects. Since none of these interpretations has offered a sufficient explanation of the objects of dianoia, I conclude that a new approach is needed.

2 Argument for New Alternative: Mental Images as Objects of Dianoia

Below, drawing from insights arising in the debate, I will argue for a new position that avoids all three objections. Though I agree with proponents of the intermediates that mathematicians are thinking about intelligible objects other than Forms, I agree with the sensible things interpretation that the objects of dianoia are on the same ontological level as the objects of pistis. I posit that dianoetic reasoners use and think about mental images of the Forms. My argument involves three steps. First, I shall argue that the summoning process integral to dianoetic reasoning leads the soul to separate out Forms and develop

the same ratio as are eikasia and pistis, we may infer that the objects of dianoia are likenesses of the Forms.

a dim and incomplete awareness of them as universals separate from their instances. Second, I shall argue that, as a result of further summoning, mathematicians construct idealized mathematical objects by hypothesizing how Forms would manifest themselves under particular conditions. Third, I conclude that these hypothesized entities are mental images of the Forms existing at the same ontological level as do the objects of pistis. I end with brief comments on the advantages of this interpretation.

2.1 Summoning and Awareness of the Forms

Summoning is basic to the mental condition of dianoia. In Book 6, Socrates places mathematics in dianoia, the lower level of the intelligible section of the divided line. In Book 7, lines 521d-522e, he leads his interlocutors in a search for the subject that will turn the thinking soul from the realm of becoming to that of being. When they cannot identify a particular subject, Socrates suggests that they consider one that touches all subjects, and he directs them to number and calculation. “But,” Socrates says, “no one uses it correctly, namely as something that is really fitted in every way to draw one towards being,” or as a summoner (523a1-3). Only after establishing that calculation and arithmetic summon in lines 524e-525a does Socrates confirm that they are to be included in his curriculum of subjects that turn the eye of the soul (525b).

At 523b-c Socrates distinguishes summoners (παρακαλοῦντα) as sense perceptions that suggest opposite characteristics at the same time. To illustrate, he gives the example of a finger (523b-524d). A finger does not summon. However, if one holds out three fingers, they may function as a summoner. Perception of a finger’s bigness or smallness is contextual and based upon its relation to others: the index finger, which is larger than the thumb, is smaller than the middle finger. When perception presents the finger as both large and small, the

33 L. Franklin (2012) also concludes that mathematicians create the ideal particulars with which they work, though he terms these entities “intermediates.” For his argument, see pp. 497-505. For ways in which my position differs from Franklin’s, see below.

34 Though L. Franklin (2012) acknowledges that the mind-dependent objects of mathematics are particulars participating in the Forms (505), he does not commit to placing them at the same ontological level as sensible participants. In addition, his account of Plato’s ambivalence toward intermediates suggests that Franklin would place these ideal participants at a level between the Forms and the sensible participants. He writes that Plato “is disinclined to say that these items simply fail to exist for much the same reason one might hesitate to say that a reflection does not exist. At the same time, though, they do not fit neatly into either of the two main categories of his metaphysics: they are neither sensible particulars nor Forms, though they bear some resemblance to both categories” (505). Franklin compares their status to that of the soul, “another entity whose status is sometimes murky in the Platonic view” (505).
soul is puzzled. According to Socrates, “the soul, summoning calculation and understanding, first tries to determine whether each of the things reported to it is one or two” (524b4-5). The soul will reason that, since there is a contradiction, two things are present. And, given that there are two, each is a separate one (524b). Socrates says that, due to being summoned, “understanding was compelled to see bigness and smallness, too, not mixed up together, but distinguished—the opposite way from sight” (524c7-8). This separation has significant consequences. Socrates points out that these sorts of cases first lead us to ask what bigness is and what smallness is. He explains: “And because of this, we called the one the intelligible and the other the visible” (524c).

Comparison of Socrates’ example with a parallel passage in Book 5 suggests that these entities, separated by reason, are Forms.35 In 475e-476a, Socrates seeks to explain the objects studied by the true philosophers. A line of reasoning very close to that provided in 524b of the summoner passage appears in his conversation with Glaucon:

Since the beautiful is the opposite of the ugly, they are two
Of course.
And since they are two, each is one?”
I grant that also.
And the same account is true of the just and the unjust, the good and the bad, and all the forms. (475e9-476a5)

Since Socrates, earlier in the Republic, used this reasoning to distinguish Forms, the reader, encountering the same reasoning in Book 7, assumes that the essences separated out by summoning are Forms.

Though I do not maintain that the summoned soul enjoys the clear grasp of Form experienced in noêsis, I do argue that it is, to some degree, aware of the contrast between the sensible particulars and the universal grasped in thought. Socrates says that the summoning process leads the soul to ask “What is X?” (524c, 524e). The soul’s ability to formulate this question on its own indicates recognition of the existence of a universal separate from its instances.36

35 Franklin (2012) 487-488 opposes this view. He argues that Forms are not separated out in the lower level summoning process initiated by sense perception. See especially note 14 on p. 488.

36 For a contrasting view of implicit awareness of the Forms, see L. Franklin (2012) 493 on the mathematicians’ implicit knowledge of Forms and L. Franklin (2005) 295 and 308 on the implicit knowledge of Forms involved in ordinary discourse. Franklin argues that the mathematicians’ implicit grasp of Forms involves no awareness of Forms as such but consists merely in unreflective predication of properties.
Though interlocutors in the Socratic dialogues pursue the “What is X?” question with no apparent awareness of the existence of universals, they do not think to raise the question themselves but instead attempt to answer Socrates. Indeed, Socrates has difficulty in making the question conceptually understandable to them.

The summoned soul also has some awareness that the universal is different in kind from its instances. Socrates says that summoners awaken the soul (523d8-e1). While in this state of heightened awareness, the soul is confronted with an important contrast between the sensible particular giving rise to a summoner and the Forms separated out by the understanding. In ordinary experience, the soul might unreflectively predicate largeness of a finger at one time and smallness of the same finger at another. Comfortable in its sense experience, the soul would remain unaware that it draws upon the Forms in making predications. In contrast, when a soul is summoned, it is driven to seek answers and its understanding wakes up and takes notice. In Socrates’ example of the fingers, the soul is confronted with the fact that particulars grasped through sense perception display opposite qualities, whereas the opposites separated out by thought never appear to be their own opposite. Indeed, this crucial difference between the two types of entities accounts for how thought is able to resolve the summoner problem. Since the soul is in an awakened state during this process, I conclude that it recognizes a significant difference between the objects grasped by perception and the ones grasped by thought: the intelligible objects are a new type of entity capable of serving as an explanatory principle.

2.2 Mathematicians Hypothesize Idealized Mathematical Objects as Images of Forms

Socrates’ descriptions of the correct study of mathematics illustrate how visible things summon the soul to hypothesize mental images of the Forms. The most explicit example concerns arithmetic. In a conversation in 524b-525a,
Socrates and Glaucon conclude that the subject of number and calculation should be legislated for study by future philosopher rulers because the number one is a summoner. They both agree that visual perception of the one always at the same time suggests many. Socrates gives an example in *Parmenides* 129c-d and *Philebus* 14c-e that is relevant to this passage.40 A man appears to be one, but at the same time he appears to be many: he has a front, a back, arms, and legs. Plato is concerned that we can ascribe different numbers to an object or group depending upon what we accept as a “one” or a countable unit.41 Concluding his summoner example at 524e5-525a2, Socrates says that “[t]he soul would then be puzzled, would look for an answer, would stir up its understanding, and would ask what the one itself is. And so this would be among the subjects that lead the soul and turn it around towards the study of that which is.”

Though the summoner has brought to mind awareness of the Form of the One, the soul is not explicitly thinking of the Form when it counts. When the soul inquires into the nature of the One, it does not focus on the One itself but directs its question back toward the realm of becoming, asking its question in the context of plurality. The soul asks what constitutes a countable unit. In other words, the soul is concerned with what is required in order for reliable counting to take place.42 At 526a1-5, Socrates gives the reader insight into how mathematicians have solved the problem. He asks: “Then what do you think would happen, Glaucon, if someone were to ask them: ‘What kind of numbers are you talking about, in which the one is as you assume it to be, each one equal to every other, without the least difference and containing no internal parts?’” Glaucon replies: “I think they’d answer that they are talking about those numbers that can be grasped only in thought and can’t be dealt with in any other way” (526a6-7).43 Though the “one” of which the mathematician speaks is an intelligible entity, it is not Oneness itself but a one among many.44 At 476a, Socrates distinguishes Forms from their participants on the basis of plurality: “Each of them is itself one, but because they manifest themselves everywhere in association with actions, bodies, and one another, each of them appears to be many” (476a).

40 I. Robbins (1995) cites these examples on p. 361.
41 See J. Annas (1975) 161.
42 J. Annas (1975) makes a similar point on pp. 161-162.
43 Compare *Republic* 526a to Plato’s similar point at *Philebus* 56e.
44 For authors emphasizing this point, see J. Adam (1902) 160 and J. Klein (1968) 77-78.
I argue that the objects of thought used by mathematicians are hypothesized entities serving as mental representations of the Forms. In his discussion of the divided line, Socrates says:

I think you know that students of geometry, calculation, and the like hypothesize the odd and the even, the various figures, the three kinds of angles, and other things akin to these in each of their investigations, as if they knew them. They make these their hypotheses and don’t think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. (510c2-d1)

In Socrates’ examples, mathematicians are hypothesizing not propositions but things—the odd and even, the various figures, and the three kinds of angles. These things are not Forms. Though mathematicians have implicit awareness of Forms, they are not interested in investigating Forms themselves but instead seek to apply this grasp of properties to particular sets of circumstances in order to solve problems. The questions posed by mathematicians arise within the context of particular mathematical disciplines. For example, plane and solid geometry study lengths, areas, and volumes, while astronomy studies solids in motion with focus on spatial and temporal relations.

In order to solve problems, mathematicians must ask themselves how Forms would appear under specified conditions. In answering such a question, the mathematician hypothesizes that the Form (F) would manifest itself in a certain way (f) under set conditions (x). The hypothesized entity exists as an object of thought. In Socrates’ example, when a summoner appearing to be both one and many puzzles the arithmetician, she is challenged by the problem of counting. Using her implicit awareness of the One (F), she separates oneness from the mixed perception and considers how the property would manifest itself within the context of the ideal, structured multiplicity described by arithmetic (x). Then, in order to perform the operations of arithmetic, she hypothesizes the existence of units (f), each equal, indivisible, and indistinguishable from others.


46 Characterization of disciplines is from M. Miller (2007) 320.

47 For instance, if circularity were to manifest within two dimensions as a non-material spatial figure with a radius of 6 cm, figure f would exist. However, since these conditions do not obtain, figure f exists only as an object of thought.

48 I believe that this view is supported by the Philebus. Contrasting the arithmetic of the philosophers to that of the common people, Socrates says that though the common people
The hypothesized entity is itself an appearance of the Form.\textsuperscript{49} In the \textit{Republic}, Forms are related to their appearances as originals to images on an analogy with sensible things. When Socrates introduces the distinction between Forms and their participants in Book 5, he depicts participants as manifestations or appearances (476a6-7) and describes them as likenesses of the Forms (476c2-7).\textsuperscript{50} Socrates’ examples of connections between sensible originals and their images help us understand this relationship.\textsuperscript{51} For example, Socrates speaks of reflections in water. Consider the case of a man’s reflection in water. When distinguishing this reflection from another (as, for example, the woman’s beside him), we may refer to it as “the man.” However, the reflection is not itself a man.\textsuperscript{52} This appearance on the surface of water is just an image, dependent upon the original for its existence and characteristics. The Form is a “real” F because of the way in which F is predicated of it. F is predicated of the Form in the sense that the Form F is what it means to be F, or, in other words, the Form F is the nature of F-ness. In contrast, F is predicated of the image in the sense that something which is not itself a real F is characterized as F due to the relationship of participation.\textsuperscript{53}

After hypothesizing entities to represent the Forms in their investigations, the mathematicians confuse images with originals in two ways. First, the mathematician mistakes an intelligible mind-dependent being for one that is mind-independent. Thought, hypothesizing that F would manifest as f in circumstance x, creates f. Since x does not obtain outside of thought, f exists only as an object of the mind. However, the mathematician assumes that f is a mind-independent intelligible being, treating it as if it were a Form. Second, the mathematician treats f as if it were the F itself by failing to acknowledge principles higher than her hypotheses. Following his examples of the odd and even, the three kinds of angles, and the various figures, Socrates says: “They make these their hypotheses and don’t think it necessary to give any accounts

\textsuperscript{49} Though the geometer’s idea of f is not a representation of some mind-independent f in the world, it is a representation of an image of F. As I hypothesize f, I am thinking of a manifestation of F.

\textsuperscript{50} The ratio of the divided line (509d-510a) and the third level of the cave allegory (515e-516a) suggest this same relationship between Forms as originals and participants as images.

\textsuperscript{51} See 509e-510a, 514b ff., and 515e-516a.

\textsuperscript{52} As R. Patterson (1985) 20 points out, the contrast between the original and its image is one between the “real or true or genuine F and something that is called ‘F’ rather than ‘G’ even though it is not a real or true F.”

\textsuperscript{53} Images bear properties in this way regardless of their medium (i.e. sensible or mental).
of them, either to themselves or others, as if they were clear to everyone (510c6-d2).” After positing the objects of mathematics, mathematicians do not inquire into the manner of their existence or the grounds of their reality but, regarding these objects as first principles, deduce conclusions from them (510d, 511a). Mathematicians, in their procedure, fail to provide explicit recognition of Forms as the ontological grounds of the mathematical objects they hypothesize. In doing so, they treat the mathematical objects as the grounds of their own reality, thus confusing them with Forms.

This interpretation is supported by Socrates’ criticism of dianoetic reasoners as dreamers. In Book 5, in the context of distinguishing the philosopher from the lover of sights and sounds, Socrates explains the difference between sleep and waking. He asks: “Isn’t this dreaming: whether asleep or awake, to think that a likeness is not a likeness but rather the thing itself that it is like?” (476c5-7). Philosophers, who can recognize sensible things as images of the Forms, are awake, whereas lovers of sights and sounds, who mistake sensible things for originals, dream. At 533b8-c3 Socrates again appeals to the sleep metaphor. Describing dianoetic reasoners, he says: “while they do dream about what is, they are unable to command a waking view of it as long as they make use of hypotheses that they leave untouched and that they cannot give any account of.” At first glance, this comment is perplexing. In the divided line passage, Socrates distinguishes dianoetic reasoners from dialecticians based upon two criteria: their use of images and their deduction from unexamined hypotheses. Since dreaming, in Book 5, is defined as mistaking image for reality, we would expect Socrates to call mathematicians dreamers in respect to the former rather than the latter. However, on my interpretation the connection is obvious. Mathematicians take their hypothesized entities to be originals and do not recognize them as images. In making this mistake, they are not looking for the right explanations of these entities and therefore lack understanding. Though the dianoetic mathematicians, unlike someone confined to the realm of opinion, can recognize visible things as images (510de), they cannot yet recognize the ideal particulars that are the subject of mathematical operations as mere images of the Forms.

2.3 Ontological Level of Mathematical Objects

My contention that mental images of the Forms, while having greater clarity than sensible ones, share the same ontological level is supported by the text. First, the claim that the objects of pistis and dianoia reside on the same ontological level is suggested by the proportions of the divided line, the ontology presented in lines 596a-598d of Book 10, and the cave allegory.
As has been much noted, any attempt to follow Socrates’ directions in drawing the divided line leads to equality in length in the middle segments, those representing dianoia and pistentis. I, along with some of the proponents of the sensible things as images of the Forms interpretation, believe that this equality in length indicates equality in ontological level. In order to make this argument, however, I must address a challenging objection. Nicholas Smith and Richard Foley argue that one cannot use the equal length of these line segments to support ontological equality while at the same time recognizing greater epistemological clarity in dianoia than in pistentis. After all, at 511e1-3 Socrates instructs his interlocutors to arrange the epistemological states “in a ratio and consider that each shares in clarity to the degree that the subsection it is set over shares in truth.” However, Socrates explicitly treats dianoia as having greater epistemological clarity than does pistentis. At 511d4-5 he describes “thought being intermediate between opinion and understanding,” and at 533d5-7 he says that the dianoetic sciences “need another name, clearer than opinion, darker than knowledge. We called them thought somewhere before.” Epistemological superiority of dianoia is implied by its belonging to the part of the line called knowledge while pistentis belongs to the part called opinion (533e) and by the fact that the third level of the cave is illuminated by sun rather than by firelight (516a).

I have two replies to this. First, according to the general reasoning of the objection, equality of length in pistentis and dianoia, the epistemological superiority of dianoia over pistentis, and the correlation of both degree of reality and degree of clarity with length of segment cannot all be held consistently. Strong arguments have been presented against the claims that equality of length in segments is an insignificant mistake on Plato’s part, and evidence that he viewed dianoia as superior to pistentis in epistemological clarity is plentiful. If the equality of the segments is not accidental, and if the equality does not pertain to degrees of epistemological clarity, it is highly likely to apply to ontology. I avoid contradiction by questioning the assumption that length of line is correlated with both ontology and epistemological clarity.

54 For a thorough treatment of the issue, see R. Foley (2008).
Socrates himself signals that this assumption deserves further consideration. Despite having set out ratios between the subsections of the divided line, at 534a5-8 Socrates says:

But as for the ratios between the things these are set over and the division of either the opinable or the intelligible section into two, let’s pass them by, Glaucon, lest they involve us in arguments many times longer than the ones we’ve already gone through.

His statement is odd. As Richard Robinson observes, if the proportions regarding the objects were the same as those of the mental states, it seems that Socrates could quickly acknowledge the fact. Instead, his statement at 534a suggests that the proportions differ, provoking the reader to contemplate how and why.

Second, the allegory of the cave, when viewed in light of Socrates’ three-level distinction between Forms, images of Forms, and copies of images of Forms in Book 10, supports my claim that these mental images of the Forms, though superior in clarity, belong on the same ontological level as the objects of ἀπαθή. In lines 596a-598d Socrates distinguishes between three kinds of beds. There is the Form of the bed, which is “the being of a bed” (597a2). The carpenter makes a bed, which both is and is not, for, according to Socrates, “if he doesn’t make the being of a bed, he isn’t making that which is, but something which is like that which is, but is not it” (597a4-5). The painter makes an imitation of a bed “third from the King and the truth” (597e7). Remember that, in the divided line, ontological levels were ranked according to their degree of truth (511e). In Book 10, based upon degrees of truth, or steps of removal from the Forms, Socrates recognizes three ontological levels corresponding to the objects of noêsis (Forms), the objects of ἀπαθή (copies of the Forms), and the objects of eikasia (copies of copies of Forms). Moreover, as argued by Bedu-Addo and others, there is reason to think that these same three ontological levels underlie the cave allegory. For example, Bedu-Addo contends that Plato includes only three grades of reality: (1) physical objects representing the objects of noêsis, (2a) images of physical objects (shadows and reflections in water) representing the objects of dianoia, (2b) images of physical objects

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58 R. Robinson (1953) 193.
60 The imitation of a bed is at a third remove because the Greeks counted the first in a series, in this case, the Form. See Grube and Reeve (1992) 258 n. 10.
(statuettes, puppets) representing the objects of *pistis*, and (3) images of images of physical objects (shadows on wall) representing the objects of *eikasia*.61

Though I incorporate reasoning from the sensible things interpretation, my conclusion diverges. Bedu-Addo and others in his interpretative camp maintained that the same objects resided at *pistis* and *dianoia*, distinguished only by the way they were viewed. I agree that the dianoetic soul recognizes that it is using sensible things as images, and that objects of *pistis* can also be objects of *dianoia* viewed under a different description.62 However I, in contrast, hold that *dianoia* also has its own unique type of object—mental images of the Forms viewed as originals. Though the images in *dianoia* have a greater degree of clarity (as is represented by their being seen in daylight), they are images of the same things as are the images in *pistis* and thus are at the same level of reality.63

One might object that the greater clarity of mental images implies greater ontological value based upon *Republic* 478e-479d. In this passage, the Form has higher ontological value than the sensible thing because it is always F whereas the sensible thing is F and not F. Though the Form “always remains the same in all respects,”64 its many sensible participants change characteristics dependent upon their contexts. For example, each of the beautiful things also appears ugly, each of the just things also appears unjust, and each of the doubles also appears half (479a-b). Because of this context variance, Socrates places sensible things between what purely is and what is not and consigns them to the realm of opinion (479c-e).

One way mental images have greater clarity than sensible ones is that they are context invariant.65 I have argued that mental images are hypothesized in

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62 Fine (1999) 231, 235-6 presents a content analysis of the line where sections are not individuated by unique objects but by their cognitive content or type of reasoning. Though I do not reject an objects view, I agree with Fine that the same object can be viewed from the perspective of more than one mental condition.

63 Mental images differ from sensible ones in that sensible images are mind-independent while mental images exist only as long as they are being thought. Shouldn’t this greater transience affect ontology? In the cave allegory, too, some objects have greater transience. A statue will continue to exist even if the original is no longer present, but a reflection in water exists only as long as the original stands before it. Since, as argued above, the three ontological levels explicitly outlined in Book 10 apply to the four types of objects in the cave allegory, and since statues and reflections are grouped together under this system, I conclude that removal from forms, not degree of transience, is the consideration relevant to ontological sorting.

64 479a1-3.

an attempt to avoid the apparent contradictions presented by summoners. For example, though a sensible thing may appear to be both one and many, a unit, which is stipulated to be an indivisible one, may not. Since the unit is context invariant, one could argue that it is F rather than both F and not F and is on a higher ontological level than sensible things.

I reply that the unit, too, fits into the category of things that are and are not. The Form remains the same in three senses: (1) it exists eternally and (2) it does not change over time, and (3) it has context invariance (Symposium 211a). Though the mental image has context invariance, and though it does not change as long as it exists (the stability of the Form guarantees stability of the mental object given the same set of stipulated conditions), the mental image does not exist eternally. Whereas the Form exists “itself by itself,” images of Forms exist in something else. For instance, a drawing of a right triangle exists on paper, and a hypothesized right triangle exists in thought. Because the mental image exists only when someone is thinking, it lacks the unconditional being enjoyed by the Forms. The mental image is F at one time and not F at another, so it too falls into the category of participants that are F and not F.

3 Conclusion

The interpretation of mathematical objects as mental images of the Forms better satisfies the three criteria of a successful interpretation than do its competitors. First, in the intelligible realm the objects of dianoia are mental images at one remove from the Forms in noêsis just as, in the sensible realm, the shadows and reflections in eikasia are sensible images at one remove from the sensible things located in pistis. Second, unlike the Forms, mental images are repeatable. Mathematicians can hypothesize properties of Form F existing under circumstances x, y, and z an unlimited amount of times and perform mathematical operations. And, because the Forms ground mathematics, context invariance

66 One might object that my position on mathematical objects undermines the truth and stability of mathematics. In response, I argue that the truth of mathematics is grounded in the Forms. Forms are eternal and unchanging, so when we hypothesize the existence of Form characteristics within set conditions, the same results will always follow. See L. Franklin (2012) for a more involved argument supporting the same conclusion. Though I believe that Forms, not the mental images, are the truth-makers of theorems, I also believe that the theorems are correctly said to be true of the mental images. I do not know if Franklin agrees. He writes that “mathematical truths can be true without being true of anything” (497) and refers to mathematical particulars “of which such theorems are, or would be, true” (484, emphasis mine).
of mathematical truth is preserved. Third, this interpretation avoids adding a fourth layer of ontological entities to the Republic. A theory treating the objects of mathematics as intermediates existing within the realm of being but at a level of reality between Forms and sensible things would be a major innovation in Plato’s metaphysics, so silence on this matter, within the metaphysical and mathematical context of the Republic, seems unlikely. The mental image interpretation, on the other hand, is consistent with the three-level distinction between Forms, images of Forms, and copies of images of Forms appearing in the cave allegory, the proportions of the divided line, and the three types of objects explicitly mentioned both in the divided line passage and in Book 10. In addition, they preserve Plato’s association of “one” with being and “many” with becoming. In conclusion, the mental images interpretation presents a plausible new perspective from which to tackle problems arising from Plato’s discussion of dianoia and mathematics.

Works Cited


67 Republic 476a and 479a.


References

*Ancient Philosophy* 15, 359-391.