CHAPTER SIX

DIORISMIC THEOREMS AND THE USE OF ANALOGY:
BOOK VII OF THE CONICA

Introduction

Apollonius describes the contents of Book VII, the last of the extant books of the Conica, in two ways. In the general introduction opening Book I, he says regarding Book VII only that it concerns diorismic theorems (περὶ διοριστικῶν θεωρημάτων), or, as the phrase is often translated, “limiting theorems.”¹ In the prefatory letter to Book VII itself, however, Apollonius tells us that the book contains “... many wonderful and beautiful things on the topic of diameters and the figures constructed on them.” Apollonius adds in the latter that such things are of great use in problems involving conic sections, including those of the kind to be discussed in Book VIII. One cannot say, therefore, that the two descriptions are inconsistent. Nevertheless, one does wonder why Apollonius’s description of Book VII in the general introduction fails to mention diameters and the figures constructed on them. Conversely, one wonders why Book VII is singled out as a book concerning diorismic theorems. Indeed, Apollonius points out that propositions in Books II, III, IV, and V are also useful in the construction of diorismoi, so why should Book VII acquire a special status in this regard? To begin to answer these questions, we shall have to understand better what is meant by a diorismic theorem and the way propositions in Book VII are theorems of that kind. This, then, is the purpose of the first part of this chapter.

In the course of the discussion, we shall have to consider the relationship between diorismic theorems and diorismoi; in particular, we shall ask whether diorismic theorems must be tied to particular problems, or whether they are, rather, the kind of theorems only useful for the construction of diorismoi of the solutions of problems. To this end,

¹ Thus, for example, Talieferro translates the phrase. I. Thomas writes “theorems about the determination of limits,” which, as we shall explain later, we take to be a less felicitous translation of “περὶ διοριστικῶν θεωρημάτων.”
we shall look at some propositions in Euclid's *Elements*, as well as some other propositions in the *Conica*; we shall also, in this context, look briefly at Halley's reconstruction of Book VIII. Following that, we shall look at the *diorismatic theorems* as independent theorems and consider what they add to our actual knowledge of conic sections.

While the first part of this chapter, accordingly, concerns the subject of Book VII, the second part concerns its dominant procedure. The most salient aspect of that is the use in Book VII of what Apollonius calls the homologue. The striking presence of the homologue has been observed, for example, by Heath. However, in their desire to find "... the geometrical substitute for algebraical operations,"\(^2\) we believe that Heath and others like him miss the true significance of the homologue. For its use, as we shall show, is to be found not only in the setting out of a proportion, an *anologia*, which lends itself, more or less readily, to algebraic formulations, but also in the making of likenesses, and this involves a kind of thinking quite different from that guided by algebra. Indeed, Book VII is replete with likeness-making. And such likeness-making, starting with Apollonius's use of the homologue, reflects, in a particularly clear way, the presence of analogical thinking in the *Conica*. Thus, whereas in previous chapters we have often pointed to particular analogies occurring in Apollonius's work, in the present chapter we shall begin a discussion of analogy as a genuine mode of thinking about conic sections (the full development and completion of this discussion we shall leave to our chapter on "Conics and Circles"). For this reason, we ought to remark, if it is not already clear, that although it has been convenient to consider the subject of Book VII in one part and its procedure in another, the separation of subject and procedure is, of course, somewhat artificial; the analogical thinking represented in Book VII is yet another expression of Apollonius's geometrical way of conceiving the object of his work.

### Diorismoi and Diorismatic Theorems

As we mentioned above, Apollonius on several occasions mentions the relevance of propositions in the *Conica* to *diorismoi*, or "limits of

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\(^2\) Heath, *Apollonius of Perga*, p. cxiii. The quotation refers, specifically, to an important proportion proven in VII.6,7 in which the ratio of the squares on conjugate