THE ENGLISH ALGEBRA OF LOGIC
IN THE 19TH CENTURY

The aim of this essay is to study works and achievements of English logicians of the 19th century. We shall consider the role they played in the development of mathematical logic, in particular their contribution to the formalization of logic and to the mechanization of reasoning. We shall present and discuss first of all works of A. De Morgan, G. Boole, W. S. Jevons and J. Venn indicating their meaning and significance for the development of mathematical logic.

Works of English logicians of the 19th century grew out of earlier ideas and attempts of G. W. Leibniz, G. Ploucquet, J. H. Lambert, L. Euler. The idea of the mathematization of logic and the development of the formal algebra in the 19th century were sources of the algebra of logic established by De Morgan, Boole and Jevons. It was in fact the beginning of the mathematical logic. The old idea of a logical calculus which would enable the analysis of logical reasoning with the help of a procedure similar to the procedure of solving equations in algebra was realized.

1. De Morgan’s Syllogistic and the Theory of Relations

We shall begin the discussion of the development of logic in England in 19th century by studying the idea of quantification of the predicate.

In traditional logic (since Aristotle) the most important role was played by syllogisms. Aristotle defined them as formal arguments in which the conclusion follows necessarily from the premises. His analysis centered on a very specific type of argument. He considered namely statements of the form: all $S$ is $P$ (universal affirmative), no $S$ is $P$ (universal negative), some $S$ is $P$ (particular affirmative).
firmative), and some $S$ is not $P$ (particular negative), abbreviated later, resp., as follows: $SaP$, $SiP$, $SeP$, $SoP$ and called the categorical sentences. Aristotle observed that one can build valid schemas of inference consisting of two premises and a conclusion being categorical sentences – they are called categorical syllogisms. If we assume that every term in a syllogism stands for a nonempty class then we get that 24 of 256 possible combinations are valid inferences.

Aristotle and his medieval followers greatly exaggerated the importance of the syllogism. Nevertheless syllogism formed the main part of logic until the beginning of mathematical logic in the 19th century. Various attempts to reshape and to enlarge the Aristotelian syllogism were undertaken. Let us mention here attempts of F. Bacon, Ch. von Sigwart and W. Schuppe. The most famous of those attempts was the “quantification of the predicate” by the Scottish philosopher Sir William Hamilton (1788–1856) presented in his book *Lectures on Metaphysics and Logic* published in 1860. He noticed that the predicate term in each of Aristotle’s four basic assertions $SaP$, $SiP$, $SeP$, $SoP$ is ambiguous in the sense that it does not tell us whether we are concerned with all or part of the predicate. Hence one should increase the precision of those four statements by quantifying their predicates. In this way we get eight assertions instead of Aristotle’s four, namely:

all $S$ is all $P$, 
all $S$ is some $P$, 
no $S$ is all $P$, 
no $S$ is some $P$, 
some $S$ is all $P$, 
some $S$ is some $P$, 
some $S$ is not all $P$, 
some $S$ is not some $P$.

Using those eight basic propositions we can combine them to form 512 possible moods of which 108 prove to be valid. The usage of statements with quantified predicates allows us higher precision than it was possible before. For example, the old logic would treat “All men are mortal” and “All men are featherless bipeds” as identical in form; whereas in the new system we see at once that the first statement is an example of “All $S$ is some $P$” and the second is an example of “All $S$ is all $P$”. But there arose some problems. It was difficult to express those new statements with quantified predicates in a common speech. Without developing a really complete and precise system of notation one finds oneself forced to apply words in a clumsy and barbarous way. Hamilton was aware of it and attempted to remedy the obscurity of phrasing by devising a curious system of notation. Though it was really curious and rather useless in practice it was important for two reasons: it had the superficial appearance of a diagram and it led Hamilton to the idea that by transforming the phrasing of any valid syllogism