APPENDIX

KILWARDBY AND MODERN LOGIC

It is a sign of Kilwardby’s genius that many of his ideas call for development within the framework of modern logic. These ideas include his notion of a “natural” proposition, his use of the notion of appropriation in systematizing the mixed assertoric/necessity-syllogisms, his doubts about mixed contingency/necessity moods in the second Figure, and a comment he makes on a *pons asinorum* for modal propositions.

*Propositions expressing natural laws*

In Chapters One and Three we saw that some propositional forms used by Kilwardby require a notion of what is the case by virtue of natural laws. The natural, in this sense, is logically similar to the obligatory in being a deontic rather than an alethic modality: it does not imply actuality. But it does imply possibility: “ought” implies “can”, and this remains true when the “ought” is one of natural rather than moral law. We also saw that the natural is a modality which is preserved by conjunction: when “p” is natural and “q” is natural the conjunction of “p” and “q” is natural.

A semantics that reflects these facts can be developed along the following lines. We assume one proposition—“n”—which comprehensively expresses The Law. We define compliant worlds as worlds in which “n” is true. We do not assume that all worlds are compliant; but we do assume that there is at least one compliant world. We define “Φp” as meaning that “p” is true in all compliant worlds; and we stipulate that “It’s natural that p” is true iff Φp. What is natural is then what is necessitated by The Law “n”. But, if “p” is necessitated by “n” and “q” is necessitated by “n” then “p and q” is necessitated by “n”. Therefore, when “p” is natural and “q” is natural “p and q” is natural.

This semantics delivers two key rules that we need for the purpose of validating Kilwardby’s results.
Rule A1. \[ \frac{p \quad q}{r} \rightarrow \frac{\Phi p \quad \Phi q}{\Phi r} \]

Proof. Suppose that \( \Phi p \). Then “\( n \)” necessitates “\( p \)”. Suppose that \( \Phi q \). Then “\( n \)” necessitates “\( q \)”. But “\( p \)” and “\( q \)” necessitate “\( r \)”. So, “\( n \)” necessitates “\( r \)”. So \( \Phi r \).

Rule A2. \[ \frac{p \quad q}{r} \rightarrow \frac{Lp \quad \Phi q}{\Phi r} \]

Proof. Suppose that \( Lp \). Then “\( n \)” necessitates “\( p \)”. Suppose that \( \Phi q \). Then “\( n \)” necessitates “\( q \)”. But “\( p \)” and “\( q \)” necessitate “\( r \)”. So, “\( n \)” necessitates “\( r \)”. So \( \Phi r \).

Given our definition of “\( \Phi p \)” as “\( n' \) necessitates ‘\( p \)'”, it follows that “\( Lp \)” implies “\( \Phi p \)” The reason is that “\( Lp \)” implies that “\( n \)” necessitates “\( p \)”. So statements of necessity are a special case of statements of naturalness.

Certain statements of naturalness (namely, those of the form “Whatever can be \( b \) can be \( a \)”) imply statements of indeterminate contingency. Suppose that it’s natural that whatever can be \( b \) can be \( a \). Then “\( n \)” necessitates “Whatever can be \( b \) can be \( a \)”. Thus, since “\( n \)” is possible, it’s possible that whatever can be \( b \) can be \( a \).

An Aristotelian system of \( L/X/M \) inferences

Kilwardby is unique among the major medieval logicians in devising a system (Kilwardby\( \lambda \)) that exactly captures the Aristotelian \( L/X/M \) syllogisms, whose base comprises the LXL-1 moods, Cesare LXL, Festino LXL, Camestres XLL, Datisi LXL, Ferison LXL and Disamis XLL.\(^1\)

Of the seven major medieval \( L/X/M \) systems\(^2\) only two include or are included in Aristotle’s system. Buridan’s main system does not contain Aristotle’s, because it lacks Barbara LXL; but Aristotle’s system does contain Buridan’s, since it contains LLX-3, Darii and Ferio LXL, Festino LXL, Datisi and Ferison LXL, LXX-3, and Celarent XLX. The Kilwardby / Campsall simpliciter system (Kilwardby\( \beta \)) contains Aristotle’s system, since it is maximal. But Aristotle’s system does not con-

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\(^1\) Thom, *Medieval Modal Systems*, 12.