CHAPTER ONE

THE PORTION OF BOOK I OF THE ELEMENTS MISSING FROM MS LEIDEN 399.1 BUT PRESENT IN MS QOM 5365, ACCORDING TO THE EDITION OF RÜDIGER ARNZEN

...falling upon it.

Al-Nayrizi said: It is as if he intended the meaning that Archimedes gave, that it is the shortest distance that connects two points.

Simplicius said: Since Euclid, by his statement, “that which is equal to what is between any two points”, meant the distance that is between the two endpoints, therefore, if we fix the two points that are the extremities of the line (for he only defined the finite line in this definition), and take the distance that is between the two of them as if it were a line, even if there is no line between the two of them, that distance will be equal to the straight line which the two points terminate. And if we measure the distances that are between any two of them with a line, then we only do so with the shortest line, and that is the shortest path that is between the various things, and we do not measure them with a line that has curvature on it. And for this reason Archimedes defined it by saying, “The straight line is the shortest of the lines whose extremities are its extremities;” he meant that it is the shortest line that connects two points. Now measurement is only by means of a straight line, since it alone is definite. And this is because there is no other line like it; none of the other lines is definite, for we can connect point with point by means of bent lines and circular lines and combinations of them, some of them greater than others. And that can go on endlessly, forever.

And what is more, when Euclid had defined the genus of the line and said that it is length without breadth, he moved on to discuss its species. And the species of lines are many, and that is because they include straight lines, circular lines, and lines intermediate between straight and circular, and these are a sort of mixture of both of them. And as for those that are intermediate, some of them are lines with no order or reason to them, and therefore the geometers do not bother with them, and they are like the conic sections\(^1\), which are like the outline of the shape of beasts of burden\(^2\), like these in the picture, and like the lines
that are horn-shaped\textsuperscript{13}, and others in addition, those whose length is infinite. And they include lines that the geometers use, like the conic sections, which are the parabola, the hyperbola, and the ellipse, and the spiral lines, and...and many other lines like them, among which are marvelous things. But Euclid, for the purposes of memorization, elementary education, and measurement, defines only the straight line and the circle, which two are the most basic lines, and that is it.

And as for Plato, lo, he defined the straight line by saying, “The straight line is one whose middle conceals its two ends,”\textsuperscript{14} for if you [should place your eye] on one of its two extremities, and should want to look at the other extremity, then the middle point would conceal that from your view. So you would find that what is in the middle conceals the extremity that [is on the other side]. And as for this definition, it serves for evidence that it is not because the middle conceals the two extremities that the straight line is straight, but rather, it is because the line is straight that its middle ends up concealing the two extremities, and this is because vision operates straightly.

And others defined the straight line by saying “Lo, it is what is utmost in regularity.”

And others have defined it by saying, “The straight line is that whose parts, all of them, naturally coincide, all of them from all directions.” And as for the parts of the circle, although some coincide with others, yet their coincidence is not from all directions, for if you place the convex part of it upon another convex side of it, they will touch at one point only, as circles are tangent, and the two of them will not coincide. And if you place the concave side on the concave, the two will touch at two points, and the two will not coincide.

And others have defined it by saying, “The straight line is that which, if its two extremities are fixed, is itself also fixed and will not move from its position, like an axis.” And as for circular lines, if their extremities are fixed like poles, this does not prevent their revolving and moving from place to place, like the semicircle held at two points. And as for the straight line, lo, if we imagine it revolving and its two extremities staying put, it will not move from its position, and for this reason, others have defined it by saying, “The straight line is that which, if it is rotated upon its two extremities, will not move from its position.” And as for the circle, lo, if it rotates around one of its two extremities (that is to say, its center\textsuperscript{15}), it will not move from its position to a different one, but if it should revolve upon two points like two poles, it will indeed move from its position.