In the name of God, the Compassionate, the Merciful!

Euclid said, “Circles equal to one another are those whose diameters are equal, and the lines that are drawn from their centers to the circumferences are equal to one another.”

Heron said: This statement is clear, for if the diameters are equal to one another, then the lines that are drawn from the centers to the circumferences will be equal to one another, because each one of these lines is half of a diameter, and it is manifest to us that if the straight lines that are drawn from the centers to the circumferences are equal to one another, then the circles will be equal to one another, since the drawing of the circles is only by means of the radius that is between the centers and the circumferences, which are half the diameters.

Euclid said, “A straight line tangent to a circle is that which, if it is made to touch the circle and is extended on both sides at the same time, does not cut the circle.

“And circles that are tangent one to another are those which, if one touches the other, do not cut one another.

“Straight lines equal in distance from the center are those such that the perpendiculars going out to them from the center are equal to one another.

“And the greater of them with respect to distance from the center is the one unto which the perpendicular going out from the center is greater.”

Heron said: Lo, the mathematician wanted to explain the distance that is between centers and continuous straight lines, and for this reason he mentioned perpendiculars, and that is because it is possible for us to draw many lines from any point to any line, but as for the distance that is between the point and the line, it is the perpendicular drawn from that point to that line.

Euclid said, “And a segment of a circle is the figure that a straight line and a piece of the arc of the circumference of the circle enclose.”
“And as for the angle of a segment, if some point is identified on the arc of the segment, then the two straight lines that are drawn from it to the two endpoints of the base of the segment enclose it."\(^{L1}\)

“And if two straight lines enclosing an angle enclose an arc, then that angle\(^{B5}\) is said to stand upon that arc."\(^{L2}\)

“A sector of a circle is the figure that the two straight lines enclosing an angle\(^{B6}\) and the arc that the angle stands upon enclose."\(^{B6}\)

Heron said: By the arc he means what the angle subtends.\(^{B7}\) And the species of sector are two, namely, those whose vertices are at the centers, and those whose vertices are at the circumferences. And as for those whose vertices are neither at the centers nor at the circumferences, lo, they are not sectors; instead, they are merely similar to sectors.

Euclid said, “Similar segments of circles are those whose angles are equal to one another, or those, the angles falling within which are equal to one another.”

Heron said: It is certainly necessary for us to know that if segments of circles are similar, then the angles drawn inside them are equal, and, the converse of this, if the angles that fall in the segments are equal to one another, then those segments are similar.

And\(^{H1}\) the species of figure are these: the circle, the segments of the circle, the lens, and the lune. As for the circle, it is the figure which we already defined in the discussion of the figures that straight lines enclose. And as for a segment of a circle, it is the figure that a straight line and an arc of the circumference of a circle enclose. And if two circles intersect, then the piece common to the two of them is called a lens, and the two remaining pieces are called, each one of them, lunes.

Here end the definitions.\(^{L3}\)

\(^{B8}\)If a straight line should pass over a circle and touch it from the outside, but not cut into any part of it, it is called a tangent to the circle.

And if circles should touch one another but not cut one into the other, then they are called tangent to one another.

And if there should be lines within the circle such that the perpendiculars drawn to them from the center are equal to one another, then the distances of the lines from the center are equal; otherwise the farthest away of them is the one whose perpendicular is the longest.

And as for the segment of a circle, the straight line that encloses it is called a chord. And a part of the circumference is called an arc,