CHAPTER 2

USAGES OF "PROOF" AND "PROVING"

The words "proof" and "proving" are used in everyday life, mathematics, and mathematics education in a number of distinct ways, usually without comment. For researchers in mathematics education this can lead to confusion and may be a serious obstacle to future research (Balacheff, 2002/2004; Reid, 2005). Without trying to establish the "right" usages of these words, we will outline here some frequent ones and describe the differences between them.

As you read this chapter you may want to reflect on these questions:

- What does "proof" mean to you?
- What should "proof" mean to students in schools?
- How can you determine what an author means by "proof"?

EVERYDAY USAGES

In everyday English, "proof" and "proving" can refer to convincing someone of something, or to testing something to see if it is correct.

Convincing

When we doubt a statement, we may ask, "Do you have any proof of that? Can you prove it?" In these questions proof means evidence, and proving means convincing. When Shakespeare's Othello says, "Be sure of it; give me the ocular proof" (Act III, scene 1) he means that Iago must convince him of the truth of his accusation by providing visible evidence. What counts as convincing evidence depends on context, and may include physical force, verbal abuse, social pressure, or anything else that persuades someone else. In the Sidney Harris cartoon captioned "You want proof? I'll give you proof!" the humour comes from a shift in context, as one mathematician is shown convincing another mathematician by punching him in the nose, which is an everyday, but not a mathematical usage of "proof" as convincing.

Testing

"Prove" is derived from the Latin verb *probare*, which means to test, to try. The English verb "probe" still carries this meaning. Taking "prove" as meaning "convince" when it means "test" can lead to odd interpretations of common expressions. For example, the expression "the exception which proves the rule" is often taken in the paradoxical sense of asserting that the presence of a counterexample establishes the general truth of a rule, which follows if “prove” is taken to mean providing convincing evidence. However, the expression is not so paradoxical if “prove” is being used to mean “test”. Then saying “the exception proves the rules” amounts to suggesting that examining exceptions closely and reasoning out the way they occur can lead to a clarification and improvement of the rule. This interpretation is reminiscent of Lakatos’s (1976) process of proof-analysis in which counter-examples and proving interact to improve theorems in mathematics (see Chapters 1 and 11).

The use of “prove” to mean “test, try” can also occur in the noun form; a “proof” can be a test or a trial. In some common phrases, “proof-read,” “proof of the pudding,” “100 proof,” the word “proof” is used in this way. Words like “waterproof” and “fireproof” are also based on this meaning; they describe objects that have been tested and found to be resistant.

SCIENTIFIC USAGES

When one reads an article about a scientific discovery, one might encounter the words “proof” and “proving” used to refer to convincing, but on the basis of special types of evidence.

> Experiments *Prove* Existence Of Atomic Chain ‘Anchors’
>
> Atoms at the ends of self-assembled atomic chains act like anchors with lower energy levels than the “links” in the chain, according to new measurements by physicists at the National Institute of Standards and Technology (NIST).
>
> The first-ever *proof* of the formation of “end states” in atomic chains may help scientists design nanostructures, such as electrical wires made “from the atoms up,” with desired electrical properties. (NIST, 2005, italics added)

When scientists “prove” something they offer convincing evidence, but that evidence must be of a special type appropriate to science.

MATHEMATICAL USAGES

Godino and Recio (1997, Recio & Godino, 2001) make a distinction between two usages of the words “proof” and “proving” in two areas of mathematics: foundations of mathematics and mainstream mathematics. This distinction is similar to the distinction made by Douek (1998) between “formal proofs” and “mathematical proofs”, the distinction made by Davis and Hersh (1981) between metamathematics and “real mathematics”, and our distinction between formal proofs and semi-formal proofs which we mentioned in Chapter 1.

In foundations of mathematics, proofs give theorems “a universal and intemporal validity”, “they rest on the validity of the logic rules used,” “the use of formal languages is required,” and proving is a way of coming to grips “with the theoretical