4.1 Philosophy of Logic and Philosophical Logic

The idea of logic as a universal language is not new. Leibniz, for example, tried to create a *characteristica universalis* that, thanks to its precision, would have allowed the settlement of any argumentative dispute mechanically. Leibniz thought it would be possible to “calculate” the conclusion of any argument expressed in such a perfect language. This idea closely resembles the mathematical structure behind the methods and the results of today’s logic theories. So, it is no accident that the birth of modern logic is intimately connected with particular aspects of mathematics. George Boole developed an algebra of logic by transferring algebraic methods to logic, thus succeeding in creating a language powerful enough to build algorithms which can be applied to infinite arguments. This result overcame traditional Aristotelian logic and opened the way for the development of various and more sophisticated languages of logic, each of them connected to the hope of accomplishing Leibniz’ dream. Gottlob Frege, primarily, believed modern logic was the *characteristica universalis* and the *calculus ratiocinator*. According to Frege, logic was the foundation of mathematics.

To outline the history of modern logic from Boole to the present goes beyond the goal of this chapter. Instead, in this section we take into account a few of the most interesting aspects of contemporary logic. Our choices, necessarily limited, were driven by the fact that these theories represent well the idea of logical pluralism, a prominent feature of logic acquired since the end of the Second World War. We discuss later what we mean by logical pluralism. However, we can anticipate that the idea of logical pluralism arises from the development of the idea of a “great logic” which mostly underlies the so-called “season of foundations” which took place during the first three decades of the last century. By great logic we mean a unifying theory of inferential processes of human rationality. The failure of this project led to the creation of different logics. Such an outcome had technical as well as philosophical consequences.

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1 Though this contribution has been conceived and discussed together, Andrea Pedeferri is the author of sections 4.1, 4.3, while Francesco Berto is the author of sections 4.2, 4.4, 4.5.
A unified idea of logic entails that every aspect of the domain of the theory could be clarified and justified by the tools and laws of the theory itself. Three different philosophies of logic rose from this idea: logicism, formalism and intuitionism. However, each of these schools hold (often radically) different positions with respect to each other; differences not just about logic proper, but also on key metaphysical consequences like, for example, the ontological status of logical and mathematical objects and abstract objects in general.

Before discussing the topics in this chapter in a more specific way, we will briefly sketch the situation of the logical currents before and after the Second World War.

4.1.1 The Situation after the 1930s

The hope of building a great logic which spanned the first three decades of the last century (which was at the time essentially represented by the finitist program by Hilbert), was destroyed once and for all by Kurt Gödel's results, proving the inconsistency of one of the main bedrocks of the program. Gödel's incompleteness theorems showed essential limitations for formal systems, such as those used to axiomatize arithmetic. Accordingly, the consistency of such systems could not be proved within the systems themselves, with the result that even simple logical systems could not stand by themselves. This result clashes with the idea of a strong unity that is intrinsic in any omni-comprehensive theorization. The impact of these results was huge, also from the point of view of the philosophy of mathematics. For example, the formalist project was so badly hit by Gödel's results that for its proponents it would not have made sense to call themselves formalists. From the 1930s onwards the monolithic structure of the foundational schools started to fade, yielding a plurality of positions which identify or intertwine themselves with the plurality of logics that rose after Gödel's impact.

We take a look now to the main positions in logic and the philosophy of mathematics, starting from the three foundational schools which had, and still have, a certain degree of popularity among logicians and mathematicians.

Logicism

Logicism, especially Gottlob Frege's system, was hit by the Russell paradox well before Gödel's results. Russell (1903) tried to give a new direction to logicism with his theory of types. However, the success of Hilbert's program shadowed logicism; already by the 1920s logicism had lost most of its strength. Today, logicism is well represented by the so-called neo-fregeans (or neo-logicists) (See Hale & Wright 2001). Crispin Wright and Bob Hale are among the main scholars of the neo-fregean project. Neo-fregeans try to recover Frege's program