The Property Paradox in (Not So Plain) English*

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1 The Property Paradox in the ‘Naïve Theory of Properties’

Reference to properties is widely used in semantic analyses of natural language (e.g. Chierchia & Turner (1988); Zimmermann (1993), and Moltmann (2004), among others). But as is summarized in Field (2004), the ‘ naïve theory of properties’ is prone to a ‘property’ version of Russell’s paradox:

According to the naïve theory of properties, for every predicate \( \Theta(x) \) there is a corresponding property \( \lambda x \Theta(x) \). Moreover, this property \( \lambda x \Theta(x) \) is instantiated by an object \( o \) if and only if \( \Theta(o) \). More generally, the naïve theory involves the following ‘ naïve comprehension schema’:

\[
\text{NC. } \forall u_1 \ldots u_n \exists y [ \text{Property}(y) & \forall x (x \text{ instantiates } y \leftrightarrow \Theta(x, u_1 \ldots u_n))].
\]

This naïve theory of properties has many virtues, but it seems to have been shattered by (the property version of) Russell’s paradox. ‘Seems to’ have been shattered? There is no doubt that it was shattered, if we presuppose full classical logic. Let us use the symbol \( \in \) to mean ‘instantiates’. The Russell paradox involves the Russell property \( R \) corresponding to the

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When I first met Ede at Sinn und Bedeutung 1998 in Leipzig, he asked me: “Bist du Elsässer?”—and I didn’t quite understand the term (I had learned my German in Berlin, and in Hegel). We have had many deeper conversations in the meantime, on every conceivable topic in semantics, logic and the philosophy of language. They have been a constant intellectual treat and have provided me with long-term food for thought (not to mention the drinking). It is a pleasure to dedicate these musings to him—and to wish him many happy returns.

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Field’s enterprise is to find a logic that is strong enough to capture part of the naïve theory of properties, yet not so strong that it generates paradoxes. This is certainly a worthy logical task; but if one is interested in the semantics of natural language, one should first ask whether paradoxes can or cannot be constructed from fragments of English that only include ‘property talk’. Obviously, if we gave ourselves the word property and made the necessary assumptions to ensure that for every formula, there exists a corresponding property, we could express in plain English the property paradox. All we would need to do is to express with English words the formulas that are mentioned in the paragraph by Field that we quoted at the outset. But this should certainly not be taken to show that the grammar of English suffices to generate the property paradox. Rather, one should conclude from a natural language rendering of Field’s reasoning that certain assumptions about properties lead to paradox. Under this view, then, nothing in the grammar of English would have to be modified if we were to abandon the belief that for each formula there exists a property that corresponds to it. Thus it might be reasonable to conclude that all is well with the grammar of English, though one might get into trouble if one insists on making misguided assumptions about ontology.

The situation is more complex, however. We will now see that quantifiers and pronouns, combined with plausible rules of inference (ones that have nothing to do with the word property or any other technical terminology), might suffice to generate the property paradox in English. However, some linguistic work is needed to come up with the paradoxical statements, which are very complex, for reasons outlined below. But before we embark on this construction, we need to be clear about the role played by quantifiers and pronouns independently of the issue of property talk. Thus we will start with some brief remarks about individual-denoting quantifiers and pronouns before applying our results to property talk.

2.1 Individual-denoting Quantifiers and Pronouns
Quine famously argued that ‘to be is to be the value of a variable bound by an existential quantifier’. His criterion applies most clearly to a theory stated in