CHAPTER 8

Justification, Confirmation, and the Problem of Mutually Exclusive Hypotheses

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Abstract

According to a recent proposal (Shogenji 2012; Atkinson 2012), the degree of justification for accepting a hypothesis should be given in the form of a function of the conditional probability and prior probability of the hypothesis. Shogenji and Atkinson impose a certain condition on measures of justification, and prove that there is only one function (up to monotonic transformations) that satisfies this condition. In this paper we show that Shogenji’s function $J$ is not suitable for the purpose of expressing the degree of justification of hypotheses by evidence. We identify a particular property that the function $J$ shares with most of the known measures of confirmation, and whose possession by a function implies that it will always include a pair of mutually inconsistent hypotheses in the set of justifiably accepted hypotheses given some evidence. We formulate a general criterion that predicts which functions are prone to the problem of mutually exclusive hypotheses, and we discuss what functions can escape this problem.

1 Justification of Hypotheses

One of the most fundamental questions in epistemology is the question of when we are justified in accepting a particular hypothesis in the light of available evidence. The standard answer to this problem is based on the assumption that justification is directly related to the posterior probability of the hypothesis (calculated from Bayes’s theorem), i.e. the higher the probability the evidence bestows on the hypothesis, the greater its degree of justification should be. However, recently this popular belief has been called into question. In particular, Shogenji (2012) and Atkinson (2012) argue that the degree of justification of a hypothesis should be expressed with the help of a more complex function of posterior and prior probabilities. Shogenji and Atkinson give an explicit mathematical form of such a function, and they prove that it is unique up to ordinal equivalence. Interestingly, Shogenji’s measure of justification turns out to satisfy the usual requirements imposed on measures of confirmation. Shogenji and Atkinson show that their proposed measure of justification satisfactorily
deals with several challenges that the traditional notion of confirmation faces, such as the irrelevant conjunction problem and the conjunction fallacy problem. Thus Shogenji’s function may also be preferred as a unique (up to ordinal equivalence) measure of confirmation of hypotheses by evidence.

The fundamental assumption of Shogenji’s approach is that justification should be motivated by the dual goal of cognition, which is to increase true beliefs and reduce false beliefs. As the risk of accepting a false belief is inversely related to the conditional probability of a hypothesis given the evidence, the degree of justification should be directly related to the posterior probability of the hypothesis. This is quite an uncontroversial claim. However, the postulate of increasing true beliefs is realized, according to Shogenji, not by simply lowering the threshold on the acceptable posterior probabilities of hypotheses (in order to admit more potentially true hypotheses), but by increasing their informativeness. And because it is typically assumed that the amount of information associated with a given proposition is inversely related to its prior probability, it follows that the degree of justification of a given hypothesis should increase when its prior probability decreases. Thus a proper measure of justification \( J(e, h) \) should be a non-decreasing (i.e. increasing or constant) function of the conditional probability of a hypothesis given the evidence, and a non-increasing (decreasing or constant) function of the hypothesis’ prior probability, as explicated below:

(1.1) If \( P(h_1 | e_1) > P(h_2 | e_2) \) and \( P(h_1) = P(h_2) \), then \( J(h_1, e_1) \geq J(h_2, e_2) \),

(1.2) If \( P(h_1) > P(h_2) \) and \( P(h_1 | e_1) = P(h_2 | e_2) \), then \( J(h_1, e_1) \leq J(h_2, e_2) \).

In addition to that, Atkinson (2012, p. 50) presents a simple argument that a measure of justification should be a function of just two independent arguments \( P(h) \) and \( P(h | e) \) (it should not depend non-trivially on the probability of evidence \( P(e) \)):

(1.3) If \( P(h_1 | e_1) = P(h_2 | e_2) \) and \( P(h_1) = P(h_2) \), then \( J(h_1, e_1) = J(h_2, e_2) \), even though it may be that \( P(e_1) \neq P(e_2) \).

Clearly the above three requirements do not pick out a unique measure of justification, not even up to ordinal equivalence. For it is well known that there are numerous ordinally non-equivalent functions of confirmation which satisfy requirements (1.1)–(1.3).\(^1\) However, Shogenji argues further that a measure of justification should meet an additional condition. He defends the intuitive

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1 We will see some of these functions later in the text.