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KNOWLEDGE FOR TEACHING MATHEMATICS

An Introduction

This introduction to Volume 1 of the Handbook draws on some teachers' answers to prompts about a particular mathematics question to highlight the challenge and complexity of describing the knowledge that mathematics teachers need in order to be able to teach. It points the way to the various chapters in the volume that provide theoretical and practical perspectives on the many dimensions of this knowledge for teaching.

VOLUME 1 OVERVIEW

This volume presents research and theoretically informed perspectives on *Knowledge and Beliefs in Mathematics Teaching and Teaching Development.* The chapters together address the "what" of mathematics teacher education, meaning knowledge for mathematics teaching and teaching development and consideration of associated beliefs. As well as synthesising research and practice over various dimensions of these issues, the volume offers advice on 'best practice' for teacher educators, university decision makers, and those involved in systemic policy decisions on teacher education.

There are four sections. The first, about mathematics discipline knowledge for teaching, contains chapters on mathematics discipline knowledge from both East Asian and Western perspectives, with separate chapters addressing primary/elementary teacher education and secondary teacher education, along with a chapter on approaches for assessing this mathematics knowledge of prospective teachers. The second section describes ways of thinking about how this mathematical knowledge is used in teaching. It includes chapters on pedagogical content knowledge, on knowledge for and about mathematics curriculum structures, the way that such knowledge can be fostered with practising teachers, on a cultural analysis of mathematical content knowledge, and on beliefs about mathematics and mathematics teaching. The third section outlines frameworks for researching issues of equity, diversity and culture in teaching mathematics. The fourth section contains a description of an approach to methods of researching mathematics discipline knowledge of teachers.

This introduction is not an attempt to summarise the chapters. (I encourage you to read a perceptive description of the various chapters and their emphases, along with insights into ways of progressing thinking about knowledge for teaching, in the review chapter written by John Mason). Nor is this introduction an overview of

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the various dimensions of knowledge that teachers might be expected to have. The different sections on the volume illustrate those. Nor is it a summary of the respective research perspectives or ways of viewing knowledge for teaching. The abstracts and the respective chapters do that.

Rather, this introduction argues that the issues of teacher knowledge and belief are important and complex. It presents a rationale for anyone with an interest in mathematics teacher education to read the volume. To begin, I describe why teacher knowledge matters, then through a particular mathematics question present three perspectives on the knowledge needed for teaching mathematics, and finally consider what this means for teacher education.

TEACHER KNOWLEDGE MATTERS

The challenge of describing succinctly the knowledge required for teaching is reflected in the debate within the mathematics education community on key issues and characteristics of effective mathematics teaching. On one side of the debate, there is substantial support for a need to intertwine conventional discipline-based learning with physical, personal and social dimensions, and the imperative to develop expertise relevant for demands of global economy and the nature of thinking required. As an example, there are explicitly stated demands in Australian curriculum documents, such as: students should demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society, and develop understanding of the role of mathematics in life, society and work, as well as developing flexible and creative approaches to solving problems. Those on this side of the debate recommend that students work on questions illustrating the usefulness of mathematics and those that foster creativity and engagement. The other side of the debate takes a more explicitly mathematical perspective with attention to the principles, patterns, processes, and generalisations that have conventionally formed the basis of the mathematics curriculum. It can be assumed that proponents of this side of the issue would anticipate teachers using classroom experiences that focus students' attention onto the mathematics.

It is stressed that this debate is far from academic. Schools in Western contexts, at least, are confronting serious challenges from disengaged students (e.g., Russell, Mackay, & Jane, 2003), with the implication that more interesting, functionally relevant tasks can enhance engagement (see Klein, Beishuizen, & Treffers, 1998). At the same time, there is a serious decline in the number of students entering university level mathematics courses (at least in the countries where the debate rages), threatening international competitiveness and innovation, fostering calls for more mathematical rigour at secondary level. As society, commerce, technology and more or less everything else is becoming more complex, to understand the complexity and contribute to developments requires an understanding of mathematics, and not only the formal processes, but also the power of generalisation, the nature of problem solving, and the demands for creativity, adaptability and the on-going nature of learning.