WHO'S AFRAID OF HIGHER-ORDER LOGIC?

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1. Introduction

One of Henri Lauener's philosophical heroes is Quine, and one of Quine’s best-known views is that for regimentation it is meet and right to use first-order predicate logic and set theory, but not higher-order logic. By higher-order logic I mean any logic using higher-order quantification, and by higher-order quantification I mean any which binds a variable other than a nominal or a propositional variable. Some may want to see propositional quantification also as higher-order, but since sentences, like names, are a basic syntactic category (as distinct from a functor category) I think this terminology is inappropriate. In any case, I shall not discuss propositional quantification at all in this paper. What should philosophers think about higher-order quantification?

This paper is frankly programmatic: I do not claim to solve the problems I raise, especially in such a short compass. But I think it is good now and again to set out as clearly as one can the reasons for being stubbornly attached to what appear to be hopeless positions, in the hope that one can see the way out oneself more clearly, or that someone else will see the way out for one, or finally that the forlornness of one’s position becomes fully apparent. So be it.

As far as its pedigree goes, higher-order predicate logic is as old as predicate logic itself: Frege’s Begriffsschrift is explicitly higher-order, since function variables are bound. The Russell-Whitehead ramified type theory and the simple type theories of Ramsey, Church and others are all higher-order. The isolation of the first-order part of what was, until then, just (modern) symbolic logic began with Löwenheim in 1915, and continues with the work of Skolem, Hilbert/Bernays, Gödel, Church, and Gentzen. The do-
minance of first-order predicate logic (I shall from now on abbreviate this to ‘PL1’) since the 1930s is based on three facts:

(1) its relative simplicity, the familiarity of its model theory, and its nice logical behaviour: it is sound, complete, and compact.
(2) the success of first-order set theory as a lingua franca and foundation in mathematics.
(3) the energetic lobbying of great logicians for PL1, and their authoritative example: I am thinking in particular of Skolem, Gödel, and Quine.

But who's afraid of higher-order logic? The question is not just rhetorical. Philosophers who tend to certain philosophical points of view will find higher-order logics both necessary and problematic. I shall list the philosophical views which put such people in this uncomfortable position and show how they lead to cognitive tension. It will emerge from this who need not be afraid of higher-order logic. I shall offer reasons for holding some, but not all of these theses. Since I find myself in the uncomfortable position of those who are both attracted and repelled by higher-order logic, I shall try to set out and justify the considerations which have put my mind at least partly at rest.

2. *Structure of the Argument*

The theses which serve as point of departure are listed here. Some of them are vaguely formulated. This is intentional, since in most cases there are several variants. Each thesis will be given an proper name, here printed in bold letters.

**INADequacy**: PL1 is inadequate for several philosophical tasks.
**LOGIicism**: Logicism is an attractive and plausible thesis.
**NOSEt**: Set theory as an adjunct to logic is unattractive and unnatural.
**FINitude**: Humanly usable languages are lexically as well as syntactically finite.
**TARSKI**: A Tarskian semantics for mathematical discourse is worth striving for.