
The book under review is an anthology of texts in the contemporary philosophy of mathematics. It was published in the series of books “The International Research Library of Philosophy”. The aim of this series is to collect “in book form a wide range of important and influential essays in philosophy, drawn predominantly from English-language journals” (p. xi). Each volume deals with a field of inquiry which has received significant attention in philosophy in the last 25 years. The Library is divided into four series of volumes: (1) Metaphysics and Epistemology, (2) The Philosophy of Mathematics and Science, (3) The Philosophy of Logic, Language and Mind, (4) The Philosophy of Value.

The book under review was published as the second volume of the series “The Philosophy of Mathematics and Science”. It was edited by Michael D. Resnik from University of North Carolina at Chapel Hill. The book consists of 24 papers collected in 7 parts as well as an Introduction written by the editor and a short name index.

Introduction provides an overview of the contemporary philosophy of mathematics and a survey of the positions developed in the volume. Resnik argues here that “philosophy of mathematics became a specialty only at the beginning of the 20th century – thanks to the work of Frege and Russell” (p. xiii). This epoch brought logicism (just Frege and Russell), intuitionism (Brouwer) and formalism (Hilbert). It gradually drew to a close in the late 1940s and early 1950s. At the same time the writings of Quine (the indispensability argument), Gödel (mathematical realism) and the (later) Wittgenstein (his ideas were transmitted largely through the work of M. Dummett who applied them to intuitionistic logic) began to influence the course of the contemporary philosophy of mathematics.

The papers collected in the volume concentrate on two fundamental questions which have been in fact considered since Plato: What are mathematical objects? and How can we know them? Resnik claims that most of the collected essays were strongly influenced by articles by Paul Benacerraf and Hilary Putnam which had appeared in the decade 1965-1975, especially by Benacerraf’s “What Numbers Could Not Be” and “Mathematical Truth” and Putnam’s “Mathematics without Foundations”.

The texts in the first part of the book are collected under the label “Field’s response” and are devoted to Hartry Field’s anti-realist ideas claiming that there are no mathematical objects and that mathematics is theoretically dispensable. Field’s programme was illustrated by developing a version of Newtonian gravitation theory in a nominalistic style avoiding abstract mathematical concepts. (Resnik claims in the Introduction that “probably no other recent work in the philosophy of mathematics has generated more interest, admiration and discussion than Field’s” (p. xvii).) One finds in the first part two papers by Hartry Field (“Realism and Anti-Realism about Mathematics” and “Is Mathematical Knowledge Just Logical Knowledge?”), David Malament’s review of Field’s book Science without Numbers, Stewart Shapiro’s paper “Conservativeness and Incompleteness” in which a serious ambiguity in the notion of conservativeness – a key notion in Field’s conception – is highlighted and Field’s ability to dispense with the deductive use of mathematics is questioned as
well as John Burgess’ paper “Synthetic Mechanics” in which a discussion of the relationship between intrinsic and extrinsic foundations of physical theories is considered.

Papers collected in Part II provide another anti-realist response, namely the claim that mathematics is really talk about possibilities involving concrete objects. One can find here a paper by Charles S. Chichara (“A Simple Type Theory Without Platonic Domains”) and a paper by Philip Kitcher (“Arithmetic for the Millian”).

Papers in Parts III and IV represent the realism. In particular in Part III one finds two papers by Penelope Maddy: “Physicalistic Platonism” and “Perception and Mathematical Intuition”. The realism of Maddy is a naturalistic version of Gödel’s realism. She claims that we can see sets, more exactly, sets of concrete objects whose members are before our eyes, and that mathematics is ultimately about sets. A paper by Charles S. Chichara included into Part III (“A Gödelian Thesis Regarding Mathematical Objects: Do They Exist? And Can We Perceive Them?”) is a critical discussion of both Gödel and Maddy.

The papers by Crispin Wright (“Why Numbers Can Believably Be: A Reply to Hartry Field”) and W. W. Tait (“Truth and Proof: The Platonism of Mathematics”) included into Part IV represent another approach to realism. In particular Wright argues that to establish that numbers exist it suffices to establish that number words function as singular terms.

Part V deals with some problems connected with the usage of the second-order logic. The main question considered in papers collected in this part is whether second-order logic is a new tool for logistic and other mathematical reductionists. One finds here three papers by George Boolos (“To Be is To Be a Value of a Variable (Or To Be Some Values of Some Variables)”, “Nominalist Platonism” and “Saving Frege from Contradiction”), a paper by Michael D. Resnik (“Second-Order Logic Still Wild”) and a paper by Stewart Shapiro (“Second-Order Logic, Foundations, and Rules”). Boolos proposed to interpret second-order logic quantifiers as plural quantifiers ranging over individuals (and to avoid in this way the necessity of engaging sets, classes or Fregean concepts in concepts based on second-order logic). Papers by Resnik and Shapiro discuss Boolos’ programme.

Part VI contains papers devoted to the structuralism – the claim that mathematics studies structures and that mathematical objects are featureless positions in these structures. This view was in fact proclaimed already by Dedekind. Structuralist ideas can be found in the writings of Hilbert, Bensays, Bourbaki and category theorists. In the volume one finds papers by contemporary representatives of structuralism, namely by Michael D. Resnik (“Mathematics as a Science of Patterns: Ontology and Reference” and “A Naturalized Epistemology for a Platonist Mathematical Ontology”), Stewart Shapiro (“Structure and Ontology”) and Geoffrey Hellman (“Modal-Structural Mathematics”). A critical survey by Charles Parsons (“The Structuralist View of Mathematical Objects”) was included here as well.

The book is closed by Part VII where a paper by Charles Parsons (“Mathematical Intuition”) and a paper by Richard Tieszen (“Phenomenology and Mathematical Knowledge”) devoted to mathematical intuition were included. Parsons claims in his essay that some mathematical objects are not featureless positions in patterns and that consequently we may grasp some of them through sensuous intuition. Tieszen expounds a theory of mathematical intuition that combines themes from both Parsons and Emdud Husserl.

So far the contents of the book. One should add that the volume was published in a careful way – all papers are reproduced in an genuine setting, with genuine