The topic of my paper is Bolzano’s method of variation. Introducing and explaining this method to the present audience would mean carrying coals to Newcastle. Instead I have prepared three puzzles concerning Bolzano’s method of variation. For the first of the three puzzles I am indebted to Mark Siebel’s M.A. thesis (1996). The second puzzle has its origin in an example Quine (1960) mentions in his article “Carnap and Logical Truth” which he contributed to the Schilpp volume on Carnap. The third puzzle arises from a thesis which Bolzano’s pupil Franz Příhonský (1850) presented in his booklet *Neuer Anti-Kant*.

My paper is divided into five parts: a short introduction, puzzle one, puzzle two, puzzle three, and a final remark or quiz.

1. *Introduction*

Let me assume that we know what a Bolzanian idea-in-itself and a Bolzanian sentence-in-itself is – although and because we know exactly that we do not know it. From now on ‘idea’ will be short for ‘idea-in-itself’ and ‘sentence’ will be short for ‘sentence-in-itself’. Let me introduce the following symbols:

‘\( V \)’ for the set of ideas, and ‘\( v \)’, ‘\( v_1 \)’, ‘\( v_2 \)’, ..., ‘\( u \)’, ‘\( u_1 \)’, ‘\( u_2 \)’, ... and ‘\( w \)’, ‘\( w_1 \)’, ‘\( w_2 \)’, ... for its members,

* The style of presentation has not been changed for publication, but I have included the reactions of the audience in two cases. I am indebted to Jan Berg, Alexander Hieke and Peter Simons for critical comments and valuable improvements.
'E' (with $E \subset V$) for the proper subset of simple or elementary ideas, and 'e', 'e_1', 'e_2', ... for its members,
'S' for the set of sentences, and 's', 's_1', 's_2', ... for its members,
'L' for the set of all logical objects which are either ideas or sentences, i.e. for the union of $V$ and $S$ ($L := V \cup S$),
and 'l', 'l_1', 'l_2', ... for its members.

When we do not use linguistic expressions but rather mention them, we usually put them within quotation marks. Analogously I will put linguistic expressions within brackets in order to refer to the corresponding ideas-in-themselves and sentences-in-themselves. So ['[prime number]'] will denote the idea-in-itself of a prime number and ['[7 is a prime number]'] will denote the sentence-in-itself that 7 is a prime number.

According to Bolzano all members of $L$, i.e. all ideas and sentences, can be analyzed into parts, whereby such an analysis always ends with simple parts, i.e. members of $E$. Every member of $L$ is therefore constructed out of members of $E$ by means of certain formation principles. The set of simple ideas out of which a logical object $l$ is constructed is called its content $C(l)$ which can therefore be defined as follows:

$\text{(D1) } C(l) := \{v \mid v \text{ is a part of } l \land v \in E\}$

Bolzano never stated his formation principles explicitly, but he might have done. Let us therefore assume that he really did formulate such formation rules for constructing all members of $L$ (ideas and sentences) out of members of $E$ (simple ideas).

Then we can define the concept of a formation sequence for every idea or sentence $l$:

$\text{(D2) } x \text{ is a formation sequence of an idea or sentence } l \text{ iff there are } l_1, l_2, ..., l_n \text{ such that } x = \langle l_1, l_2, ..., l_n \rangle \text{ and the following holds:}$

(a) for each member $l_i$ of $\langle l_1, l_2, ..., l_n \rangle$
   (i) $l_i$ is a member of $L$ (i.e. $l_i \in L$),
   (ii) either $l_i$ is simple (i.e. $l_i \in E$) or $l_i$ results from members preceding $l_i$ in $\langle l_1, l_2, ..., l_n \rangle$ by means of a formation principle,