FREGE ON CONCEPTUAL AND PROPOSITIONAL ANALYSIS

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Summary
In his *Foundations of Arithmetic*, Frege aims to extend our a priori arithmetical knowledge by answering the question what a natural number is. He rejects conceptual analysis as a method to acquire a priori knowledge (see section 1). Later he unsuccessfully tried to solve the problems that beset conceptual analysis (see section 2). If these problems remain unsolved, which rational method can he use to extend our a priori knowledge about numbers? I will argue that his fundamental arithmetical insight that numbers belong to concepts is based on the recognition that different sentences express the same thought. In Frege’s philosophy of arithmetic, propositional analysis does the main work. How it can do this work will be discussed in sections 3, 4 and 5. Sections 6 and 7 explore this approach further.

1. Frege on conceptual analysis and the paradox of analysis

How can we, in a rational way, arrive at an answer to the question ‘What is a natural number?’ Certainly not by observation or experiment. In his *Philosophy of Arithmetic*, Husserl proposed to answer this question by an analysis of the sense of ‘natural number’. Frege criticised Husserl by saying:

A definition is also incapable of analysing the sense, for the analysed sense is not the original one. In using the word to be explained, I either think clearly everything I think when using the defining expression: we then have the ‘obvious circle’; or the defining expression has a more richly articulated sense, in which case I do not think the same thing in using it as I do in using the word to be explained: the definition is then wrong. (Frege 1894, 199 [319])

In effect, Frege here discovers the paradox of analysis.¹ Let us call any

¹ Discovery of the paradox of analysis is usually credited to Langford (1942) and Moore (1942) or, some years earlier, to Wisdom (1934).
definition that aims to analyse the sense of a widely used and understood word like ‘number’ analytic. According to Frege, the paradox can be set up as follows:

(A1) An analytic definition is correct if, and only if, the definiendum and definiens have the same sense.
(A2) \( A \) and \( B \) are synonymous & (\( S \) grasps the sense of \( A \) & \( S \) grasps the sense of \( B \)) \( \rightarrow \) \( S \) immediately knows that \( A \) and \( B \) have the same sense.

Therefore: (K) An analytic definition is either trivial or false.

The paradox arises for analytic definitions of general terms. It also arises for a kind of analysis that is sometimes called ‘propositional analysis’. Russell’s theory of definite descriptions is an example of propositional analysis. Russell held that definite descriptions are ‘incomplete signs’: unlike a general term, a definite description cannot be defined; one can only provide a paraphrase of a sentence containing a definite description that eliminates the definite description. If the paraphrase is supposed to preserve the sense of the original sentence and to clarify it, the paradox of analysis also arises for propositional analysis. I will come back to this point in the second half of the paper.

Frege’s first premise (A1) is independently plausible: in an analytic definition, one doesn’t want to give a new sense to a word that is already in use. Loosely speaking, one aims to clarify the sense it already has.

Frege’s second premise is essential to his theory of sense and reference. His criterion of thought identity assumes that one cannot grasp the sense of synonymous expressions without knowing that they are synonymous:

Now two sentences \( A \) and \( B \) can stand in such a relation that anyone who recognizes the content of \( A \) as true must thereby recognize the content of \( B \) as true and, conversely, that anyone who accepts the content of \( B \) must straight-away accept that of \( A \). (Equipollence). It is here being assumed that there is no difficulty in grasping the content of \( A \) and \( B \). (Frege 1906, 197 [213])

If \( A \) and \( B \) have the same sense and one could grasp the sense of both without recognising their sameness, one could rationally have different attitudes towards the thoughts expressed. For example, I could assent to ‘John is bachelor’ and reject ‘John is an unmarried man’ without thereby becoming liable to criticism. If this were the case, the possibility of having different attitudes towards what \( A \) and \( B \) say would no longer track