NUMBERS AS ONTOLOGICALLY DEPENDENT OBJECTS
HUME’S PRINCIPLE REVISITED

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Summary
Adherents of Ockham’s fundamental razor contend that considerations of ontological parsimony pertain primarily to fundamental objects. Derivative objects, on the other hand, are thought to be quite unobjectionable. One way to understand the fundamental vs. derivative distinction is in terms of the Aristotelian distinction between ontologically independent and dependent objects. In this paper I will defend the thesis that every natural number greater than 0 is an ontologically dependent object thereby exempting the natural numbers from Ockham’s fundamental razor.

Not so long ago, foes of abstract objects like numbers rallied behind a principle oftentimes called Ockham’s razor. According to this principle, we should welcome as few kinds of objects as possible in our ontology. Since numbers were deemed especially unwelcome, they were usually among the first to suffer the razor’s ruthless stroke. However, in recent times it has been proposed to rather adhere to a principle one could call Ockham’s fundamental razor: entia fundamentalia non sunt multiplicanda praeter necessitatem (e.g. Schaffer 2009). So inclined philosophers find no difficulty in extending a welcoming hand to all kinds of things. The only caveat is that (i) such objects be non-fundamental or derivative, and (ii) that they ultimately derive from something such philosophers regard as unproblematic. One such sense of the fundamental vs. derivative distinction can be found in the Aristotelian tradition, a tradition in which substances are characterized as ontologically independent (fundamental) objects whereas the properties that inhere in them are said to be ontologically dependent (derivative) objects.

It is the aim of this paper to make some headway towards showing that numbers are dependent objects and, thus, are not subject to Ockham’s
fundamental razor. To this end I will argue that a broadly Fregean way of conceiving of the natural numbers vindicates the following thesis:

**Dependence Thesis**

*Every natural number greater than the number 0 is an ontologically dependent object.*

The procedure will be as follows. In the first section, I will argue for a version of Hume’s Principle—Frege’s contextual definition of the cardinality operator ‘the number of’ (1884, §§ 62f.)—on which it is a 1st-level rather than a 2nd-level abstraction principle. This understanding of Hume’s Principle will be motivated by drawing attention to a hitherto unobserved flaw in Frege’s analyses of statements of (equi)numerousity, i.e. that they cannot handle collective predicates. In the second section, I will draw attention to the fact that Hume’s principle has an explanatory dimension in that the principle’s right-hand-side explains its left-hand side. In the third section, I will introduce a notion of ontological dependence that is just as serviceable when applied to the Aristotelian conception of substances as independent objects as it is when applied to the case of numbers. In the fourth and final section the Dependence Thesis will be vindicated.

1. **Two levels for Hume’s Principle**

In this section I will motivate and develop an understanding of Hume’s Principle on which it is a 1st-level (rather than a 2nd-level) abstraction principle. This understanding will serve as the first headstone of my argument for the Dependence Thesis.

(i) **HP as a 2nd-level Abstraction Principle:** What is the difference between abstraction principles of different levels? To answer this question it is helpful to first take a look at the general form of abstraction principles:

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\text{AP} \; \Sigma[\alpha_i] = \Sigma[\alpha_j] \leftrightarrow \alpha_i \equiv \alpha_j.
\]

Abstraction principles are intended to contextually define the term-forming operator \(\Sigma[\ ]\) by stipulating that sentences obtained by flanking ‘=’ with terms formed by combining \(\Sigma[\ ]\) with expressions \(\alpha_i\) and \(\alpha_j\) are to be true just in case the equivalence relation signified by the relational