Convexity of a set of subthreshold stimuli implies a peak detector

A. D. LOGVINENKO*

School of Psychology, Queen's University, Belfast, BT9 5BP, UK

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Abstract—It has been shown that for every model of detection (whether single- or multi-channel, with linear or non-linear channels, and whatever decision rule), provided that it predicts a convex set of subthreshold stimuli, there is a psychophysically equivalent peak detector made up of a collection of linear analysers followed by a maximum-output decision rule. In this paper, the equivalent peak detector representations of some widely accepted detection models are calculated. The calculations rest on a general technique for deriving, from a given model, a formula which specifies the analyser most sensitive to any given stimulus.

1. INTRODUCTION

The detection of a pattern implies that an observer is able to discriminate a two-dimensional luminance distribution over the visual field \( I(\alpha, \beta) = I_0[1 + c x(\alpha, \beta)] \) from a background consisting of homogeneous patch of light of the same spatial configuration and mean luminance (here \( I_0 \) is mean luminance; \( \alpha \) and \( \beta \) are horizontal and vertical spatial coordinates measured in degrees of visual angle; \( x(\alpha, \beta) \) is the pattern proper; and \( c \) is the pattern contrast). The contrast threshold for the pattern \( x(\alpha, \beta) \) is the lowest contrast, written \( c_{th}(x) \), at which the probability of discrimination is not less than some fixed critical value (discriminability index). The reciprocal of the contrast threshold is taken as a measure of contrast sensitivity. To be more exact, if \( s(x) \) stands for the sensitivity to the contrast of the pattern \( x(\alpha, \beta) \), then
\[
 s(x) = c_{th}^{-1}(x).
\]

It is well established that sensitivity to the contrast of a visual pattern is affected essentially by its luminance profile. For example, the contrast threshold for homogeneous patches of light has been found to depend on their size and shape (Ricco, 1877; Blackwell, 1946, 1972; Lamar et al., 1947; Brown and Mueller, 1965). Likewise contrast sensitivity to a sinewave grating is known to vary with its spatial frequency (Schade, 1956; Campbell and Green, 1965), orientation (Campbell et al.,

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*E-mail: a.logvinenko@qub.ac.uk
Figure 1. Vector representation of the threshold body. In so far as functions constitute an infinite-dimensional linear space, patterns can be considered as vectors in such a space. As patterns are to be detected against a background, a vector representing a pattern is drawn from the point corresponding to the background. When its contrast c varies from zero to infinity each pattern x specifies a set of collinear vectors \( c(x)x \) in a linear functional space. When contrast is not more than the threshold value, i.e. \( c \leq c_{th}(x) \), a vector \( cx \) belongs to the threshold body, otherwise it is outside it.

For the rest of this discussion it is convenient to consider the vector representations of the patterns to be detected. Each pattern \( x(\alpha, \beta) \) determines a ray in the vector functional space, the threshold value \( c_{th}(x) \) specifying a point on this ray (Fig. 1). All such points for all \( x(\alpha, \beta) \) constitute a manifold confining a closed subset which will hereafter be called the threshold surface and threshold body respectively (Fig. 1).

Note that the threshold surface is a surface of equal contrast sensitivity. Indeed, it follows from this definition that all the vectors lying on the threshold surface have unit contrast threshold. Besides, the threshold body—designated \( B \)—has been proved to be a unit-level set of the contrast-sensitivity function \( s(x) \) (Logvinenko, 1996), that is to say, it comprises patterns \( x(\alpha, \beta) \) such that:

\[
B = \{ x | s(x) \leq 1 \}, \quad \text{where} \quad x = x(\alpha, \beta).
\]  

In other words a pattern belongs to the threshold body whenever it cannot be discriminated from the background with a probability greater than the discriminability index.

Many psychophysical experiments have shown that the contrast-sensitivity function is too complicated to be described by a more-or-less comprehensible expression. This leads to the idea that, perhaps, a subject makes a decision in some internal space where the representation of the threshold body has a much simpler form. Such an internal representation is often supposed to be implemented by neuron-like pattern analysers (see e.g. Graham, 1989). The scalar outputs of these analysers are widely believed to constitute the dimensions of the internal space where any decision is made.

Mathematically an analyser is usually represented by the linear functional \( \varphi \). The response of this analyser to the pattern \( x(\alpha, \beta) \) can be specified in terms of its