The transducer model for contrast detection and discrimination: formal relations, implications, and an empirical test

MIGUEL A. GARCÍA-PÉREZ* and ROCÍO ALCALÁ-QUINTANA

Departamento de Metodología, Facultad de Psicología, Universidad Complutense, Campus de Somosaguas, 28223 Madrid, Spain

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Abstract—The transducer function \( \mu \) for contrast perception describes the nonlinear mapping of stimulus contrast onto an internal response. Under a signal detection theory approach, the transducer model of contrast perception states that the internal response elicited by a stimulus of contrast \( c \) is a random variable with mean \( \mu(c) \). Using this approach, we derive the formal relations between the transducer function, the threshold-versus-contrast (TvC) function, and the psychometric functions for contrast detection and discrimination in 2AFC tasks. We show that the mathematical form of the TvC function is determined only by \( \mu \), and that the psychometric functions for detection and discrimination have a common mathematical form with common parameters emanating from, and only from, the transducer function \( \mu \) and the form of the distribution of the internal responses. We discuss the theoretical and practical implications of these relations, which have bearings on the tenability of certain mathematical forms for the psychometric function and on the suitability of empirical approaches to model validation. We also present the results of a comprehensive test of these relations using two alternative forms of the transducer model: a three-parameter version that renders logistic psychometric functions and a five-parameter version using Foley’s variant of the Naka–Rushton equation as transducer function. Our results support the validity of the formal relations implied by the general transducer model, and the two versions that were contrasted account for our data equally well.

Keywords: Signal detection theory; transducer model; psychometric function; detection; discrimination; 2AFC.

1. INTRODUCTION

The psychometric function \( \Psi \) expresses the probability of success in a psychophysical task as a function of stimulus level \( x \), and it is a non-decreasing, four-parameter

*To whom correspondence should be addressed. E-mail: miguel@psi.ucm.es
function whose general form is
\[
\Psi(x) = \gamma + (1 - \lambda - \gamma) F(x; \theta, \beta),
\]
(1)
where \(\gamma\) (the false-positive or guessing rate) sets a lower asymptote, \(\lambda\) (the false-negative or lapsing rate) sets an upper asymptote, and \(F\) expresses how the probability of the underlying sensory event changes with stimulus level, a function with location parameter \(\theta\), scale or slope parameter \(\beta\), and lower and upper horizontal asymptotes at 0 and 1.

In empirical practice, \(\Psi\) is often assumed to have some convenient mathematical form, as determined by the choice of \(F\), and all of its parameters or a relevant subset of them are estimated from the data. This is justifiable insofar as the issue under research merely requires a comparison of parameter estimates across conditions. However, it should be realized that \(\Psi\) is only an observable manifestation of the internal workings of the sensory mechanism governing performance on the task, a mechanism that includes both a stimulus-processing unit and a decision operator. Then, \(\Psi\) is an empirical outcome to be explained as a result of the operation of the sensory mechanism.

The most common approach to explaining the observable \(\Psi\) is by consideration of signal detection theory (SDT). When SDT is applied to 2AFC detection or discrimination experiments, each stimulus is assumed to produce an internal response that is a random variable with a certain distribution, and the subject is assumed to respond by indicating the interval that elicited the larger internal response. The mean of the distribution of internal responses increases with input strength, and its variance may also vary with input strength. An alternative but functionally equivalent approach is considering that the internal response consists of a non-random component determined by input strength and a random noise component that may be either additive (i.e. noise variance is constant) or multiplicative (i.e. noise variance varies monotonically with input strength). The final mathematical form of the observable \(\Psi\) depends on the form and parameters of the assumed distribution of internal responses, including expressions for the functional dependence of its mean and variance with input strength. However, if these internal responses are modeled at the level of the individual units that subserve the detection or discrimination process, the final mathematical form of the observable \(\Psi\) further depends on assumptions as to how the responses of these individual units are aggregated (e.g. by probability summation or by non-linear pooling).

Theoretical analyses by Pelli (1985) and Tyler and Chen (2000) have shown how specific mathematical forms for the observable \(\Psi\) arise as a result of a particular set of assumptions, and papers have been published that have addressed empirically the issue of the validity of specific forms for these assumptions (e.g. whether noise is additive or multiplicative) but necessarily by taking for granted the validity of other untested assumptions (see Gorea and Sagi, 2001; Kontsevich et al., 2002). It is nevertheless important to realize that the conclusions reached from these empirical