Which geometric model for the curvature of 2-D shape contours?

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Abstract—We investigated the geometric representations underlying the perception of 2-D contour curvature. 88 arcs representing lower and upper halves of concentric circles, or halves of ellipses derived mathematically through planar projection by affinity with the circles, a special case of Newton’s transform, were generated to produce curved line segments with negative and positive curvature and varying sagitta (sag) and/or aspect ratio. Aspect ratio is defined here as the ratio between the sagitta and the chord-length of a given arc. The geometric properties of the arcs suggest a regrouping into four structural models. The 88 stimuli were presented in random order to 16 observers eight of whom were experienced in the mathematical and visual analysis of 2-D curvature (‘expert observers’), and eight of whom were not (‘non-expert observers’). Observers had to give a number, on a psychophysical scale from 0 to 10, that was to reflect the magnitude of curvature they perceived in a given arc. The results show that the subjective magnitude of curvature increases exponentially with the aspect ratio and linearly with the sagitta of the arcs for both experts and non-experts. Statistical analysis of the correlation coefficients of linear fits to individual data represented on a logarithmic scale reveals significantly higher correlation coefficients for aspect ratio than for sagitta. The difference is not significant when curves with the longest chords only (7°–10°) are considered. The geometric model that produces the best psychometric functions is described by a combination of arcs of vertically and horizontally oriented ellipses, indicating that perceptual sensations of 2-D contour curvature are based on geometric representations that suggest properties of 3-D structures. A ‘buckled bar model’ is shown to optimally account for the perceptual data of all observers with the exception of one expert. His perceptual data can be linked to a more analytical, less ‘naturalistic’ representation originating from a specific perceptual experience, which is discussed. It is concluded that the structural properties of ‘real’ objects are likely to determine even the most basic geometric representations underlying the perception of curvature in 2-D images. A specific perceptual learning experience may engender changes in such representations.

Keywords: 2-D contour curvature; geometric cues; psychometric function; laws of sensation; shape perception; perceptual learning.

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INTRODUCTION

Recent studies on the possible link between 2-D shape perception and the known properties of 3-D objects in the ‘real’ world have led to suggest that the perception of 2-D shape properties is largely determined by statistical relationships between geometrical characteristics of image representations and their physical sources in ‘natural’ visual environments (Howe and Purves, 2005). As discussed earlier by others (e.g. Attneave, 1954; Ogilvie and Daicar, 1967; Wilson and Richards, 1989; Morgan, 2005), the visual perception of 2-D curvature plays an important role in form perception and object recognition. Curvature may be particularly useful to the processes that allow the human perceptual system to generate 3-D representations of complex objects and scenes on the basis of simple 2-D contour images. Also, 2-D curvature has been shown to contribute to the apparent non-rigidity of objects and parts (e.g. Cavanagh and von Grünau, 1989), thereby providing ecologically significant cues for the perception of facial expressions (Lyon et al., 2000). 3-D shape representation may be enabled by local shape biases favouring symmetry or other structural regularities in the 2-D image, or by combinations of pictorial image cues (e.g. Biederman, 1987; Koenderink and van Doorn, 2003). Visual experience and learned knowledge about object properties, i.e. learned associations between specific two-dimensional projections and their correlated three-dimensional structures, may be another important factor (e.g. Biederman, 1987; Sinha and Poggio, 1996; Howe and Purves, 2005).

The 2-D geometry of curves is abundantly exploited by design engineers and architects in the conceptual design of complex spatial structures (see Fig. 1) such as spherical domes or modern free-form-design structures, so-called ‘blob designs’, in general. The use of curvilinear spaces in architecture dates back to the dawn of building shelter. Vernacular architecture is, by the nature of the materials and construction techniques used, based on curved shapes even though descriptive 2-D geometry was not necessarily referred to then for the planning and execution of building projects. It has been suggested that evolution may have produced specialized brain mechanisms that efficiently exploit the geometry of visual perceptual space (Heeley and Buchanan-Smith, 1996; Foley et al., 2004). However, neither the brain processes through which 2-D curve geometry would enable 3-D shape representation, nor which of the geometrical cues available in a simple 2-D curve would best account for its perceived quality in terms of a shape, have been hitherto identified.

The early visual processing of curved line segments has been amply investigated in psychophysical studies using two-alternative forced-choice (2 AFC) procedures for measuring visual discrimination thresholds. In such experiments, observers have to compare briefly presented curve segments and make a rapid decision regarding local differences in curvature, or curve orientation.