Quantification of local symmetry: application to texture discrimination

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Abstract—Symmetry is one of the most prominent cues in visual perception as well as in computer vision. We have recently presented a Generalized Symmetry Transform that receives as input an edge map, and outputs a symmetry map, where every point marks the intensity and orientation of the local generalized symmetry. In the context of computer vision, this map emphasizes points of high symmetry, which, in turn, are used to detect regions of interest for active vision systems. Many psychophysical experiments in texture discrimination use images that consist of various micro-patterns. Since the Generalized Symmetry Transform captures local spatial relations between image edges, it has been used here to predict human performance in discrimination tasks. Applying the transform to micro-patterns in some well-studied quantitative experiments of human texture discrimination, it is shown that symmetry, as characterized by the present computational scheme, can account for most of them.

1. INTRODUCTION

Symmetry is among the most prominent spatial relations perceived by humans. Natural and artificial objects often give rise to the human sensation of symmetry, and this sense of symmetry is so strong that the Gestalt school considered symmetry as a fundamental principle of perception. Looking around us, we get the immediate impression that practically every interesting visual area consists of some generalized form of symmetry.

We have recently presented a generalized symmetry transform and demonstrated its application to detection of interest points in natural images (Reisfeld et al., in press), in face recognition (Edelman et al., 1992) and in normalization (Reisfeld and Yeshurun, 1992) tasks. Using this measure, we have suggested a computational model that takes as an input the intensity gradient at each image point, and generates activity maps of the generalized symmetry in different scales. Areas of different texture are thus characterized by different activity on one or more of these maps. The basis of our transform is the quantification of local spatial relations between image edges in a way

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that captures the amount of symmetry support at each point. For example, parallel centered intensity gradients strongly support a symmetry point between them while collinear gradients do not. This quantification can be interpreted as an estimation of a local 'Gestalt Glue'. It computes points of interest, where attention might then be directed, e.g. eyes of a person in a portrait or heads in a group of people (Reisfeld et al., 1990). When applied more locally to images, it detects sharp corners as interest points by assigning high 'symmetry' value. Applied to texture micro-patterns, it quantifies every micro-pattern, and thus can be used as the basis for discrimination. In this paper, we present the results of applying our measure to some well-studied quantitative results in human psychophysics, and show that our simple model yields a good fit to human performance.

2. GENERALIZED SYMMETRY

In the usual mathematical notion, an object is regarded as symmetric if it is invariant to the application of certain transformations, called symmetry operations. A typical symmetry operation is the well-known reflectional (mirror) symmetry. In order to use these symmetry operations, it is necessary to know the shape of an object before we can estimate whether it is symmetric or not. However, we wish to quantify symmetry without any prior knowledge of objects, especially if the symmetry measure is used to detect regions of interest.

Our symmetry transform does not require the knowledge of the object’s shape. It performs local operations on the edges of the image. Moreover, it assigns a continuous symmetry measure to each point in the image, rather than a binary symmetry label.

We first define a symmetry measure for each point. Let \( p_k = (x_k, y_k) \) be any point \( k = 1, \ldots, K \), and denote by \( \nabla p_k = (\partial p_k/\partial x, \partial p_k/\partial y) \) the gradient of the intensity at point \( p_k \). We assume that a vector \( v_k = (r_k, \theta_k) \) is associated with each \( p_k \) such that \( r_k = \log (1 + ||\nabla p_k||) \) and \( \theta_k = \arctan ((\partial p_k/\partial y)/(\partial p_k/\partial x)) \). For each two points \( p_i \) and \( p_j \), we denote by \( l \) the line passing through them, and by \( \alpha_{ij} \) the angle counterclockwise between \( l \) and the horizon. We define the set \( \Gamma(p) \), a distance weight function \( D_\sigma(i, j) \), and a phase weight function \( P(i, j) \) as

\[
\Gamma(p) = \left\{ (i, j) \left| \frac{p_i + p_j}{2} = p \right. \right\},
\]

\[
D_\sigma(i, j) = G_\sigma(\|p_i - p_j\|),
\]

\[
P(i, j) = \left(1 - \cos (\theta_i + \theta_j - 2\alpha_{ij})\right)\left(1 - \cos (\theta_i - \theta_j)\right),
\]

where \( G_\sigma(t) = (1/\sqrt{2\pi\sigma}) \exp(-t^2/(2\sigma^2)) \) is the Gaussian, \( \sigma \) of \( D_\sigma \) corresponds to scale or a locality channel.

We define the contribution of the points \( p_i \) and \( p_j \) as

\[
C(i, j) = D_\sigma(i, j)P(i, j)r_ir_j.
\]